The Power of Time Reversal in Hamiltonian Property Testing

Richard Allen (MIT) Matthias Caro (Warwick) Yanlin Chen (Maryland)
Tom Gur (Cambridge) Angus Lowe (MIT)

STOC 2027 Submission



arXiv:2610.04996

Overview

1. Background & Problem Statement
2. Main Results
3. Proof Sketch
4. Conclusion & Future Directions

Time Reversal in Quantum Information

Time reversal is a key step in many tasks across QI

Time Reversal in Quantum Information

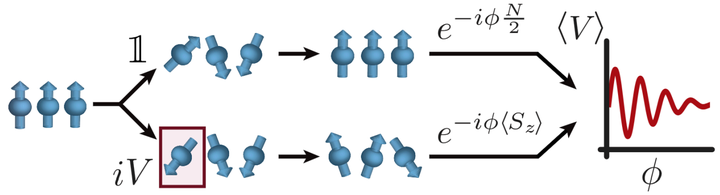
access to both e^{-iHt} *and* e^{+iHt}

Time reversal is a key step in many tasks across QI

Time Reversal in Quantum Information

access to both e^{-iHt} *and* e^{+iHt}

Time reversal is a key step in many tasks across QI

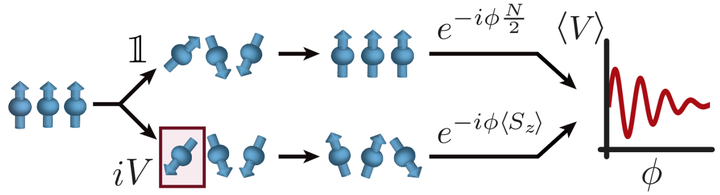


Quantum sensing [1]

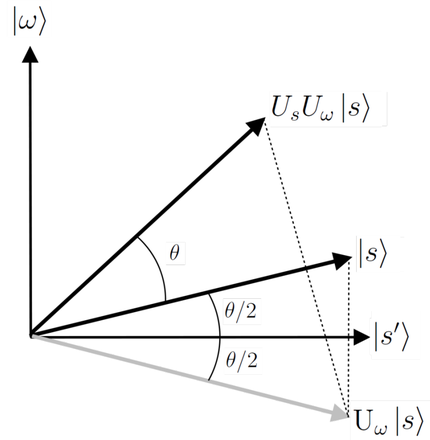
Time Reversal in Quantum Information

access to both e^{-iHt} *and* e^{+iHt}

Time reversal is a key step in many tasks across QI



Quantum sensing [1]

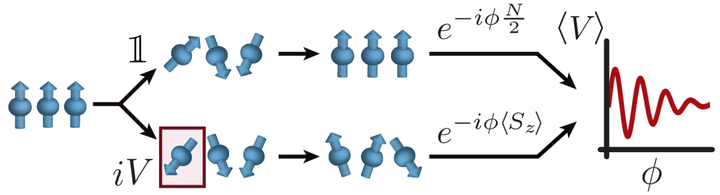


Quantum algorithms [2]

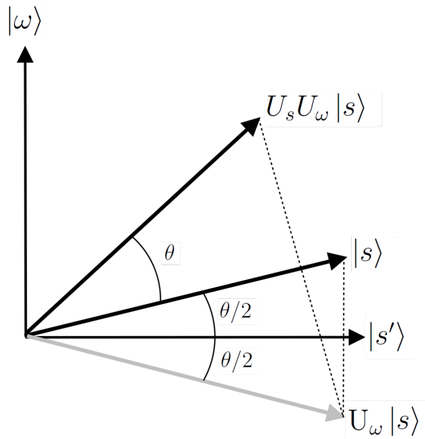
Time Reversal in Quantum Information

access to both $e^{-\mathrm{iHt}}$ *and* $e^{+\mathrm{iHt}}$

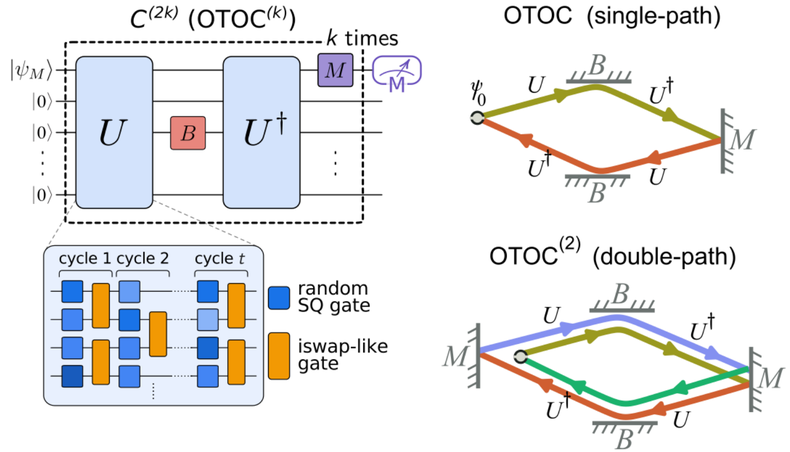
Time reversal is a key step in many tasks across QI



Quantum sensing [1]



Quantum algorithms [2]



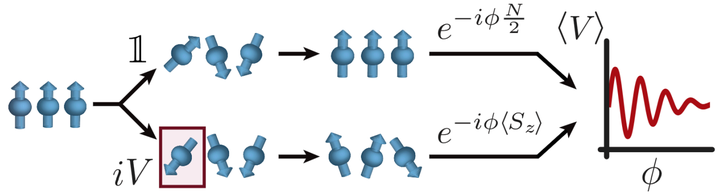
Many-body quantum systems [3]

[1] Kobrin et al. A Universal Protocol for Quantum-Enhanced Sensing via Information Scrambling.
[2] Wikipedia.
[3] Google Quantum AI. Observation of constructive interference at the edge of quantum ergodicity.

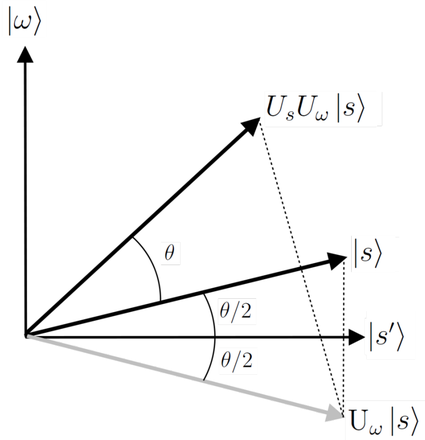
Time Reversal in Quantum Information

access to both e^{-iHt} *and* e^{+iHt}

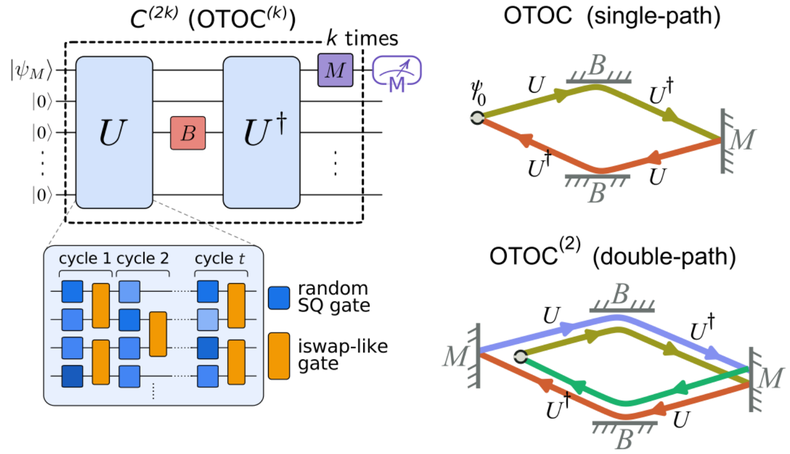
Time reversal is a key step in many tasks across QI



Quantum sensing [1]



Quantum algorithms [2]

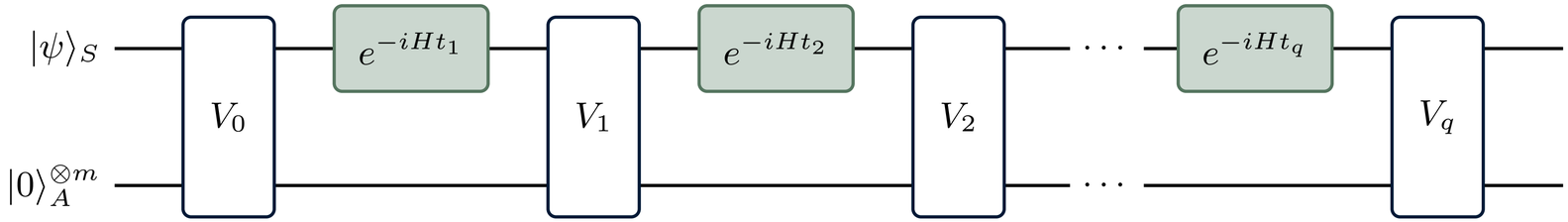


Many-body quantum systems [3]

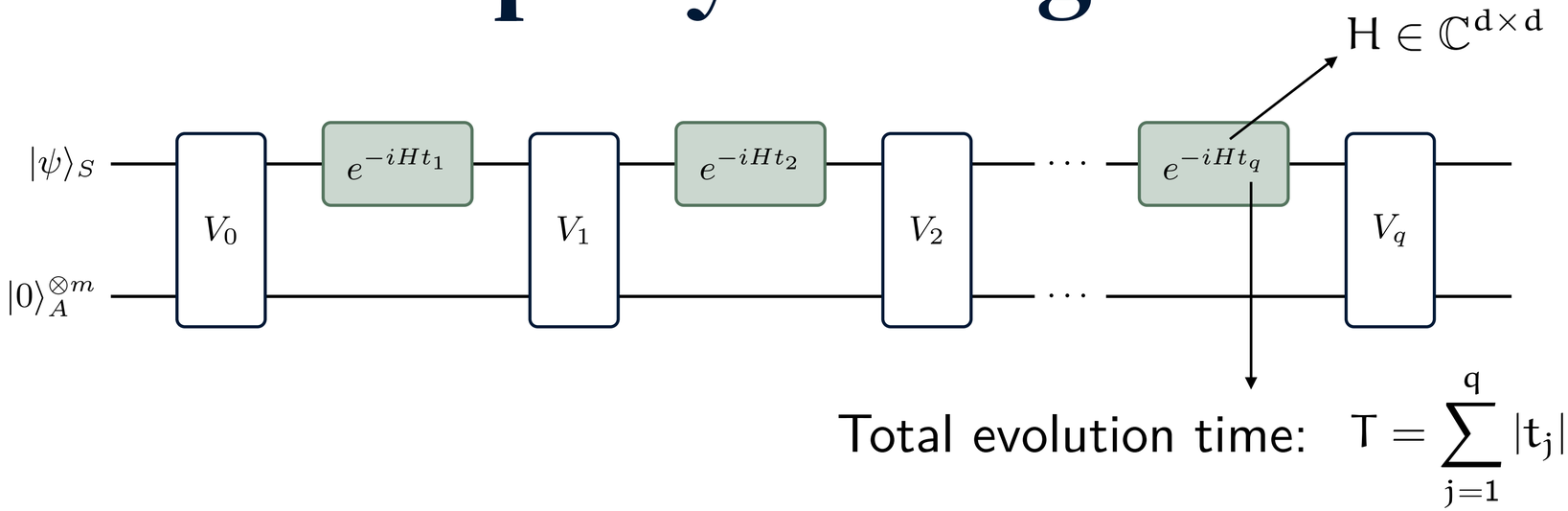
Q: *When is time reversal truly required?*

[1] Kobrin et al. A Universal Protocol for Quantum-Enhanced Sensing via Information Scrambling.
[2] Wikipedia.
[3] Google Quantum AI. Observation of constructive interference at the edge of quantum ergodicity.

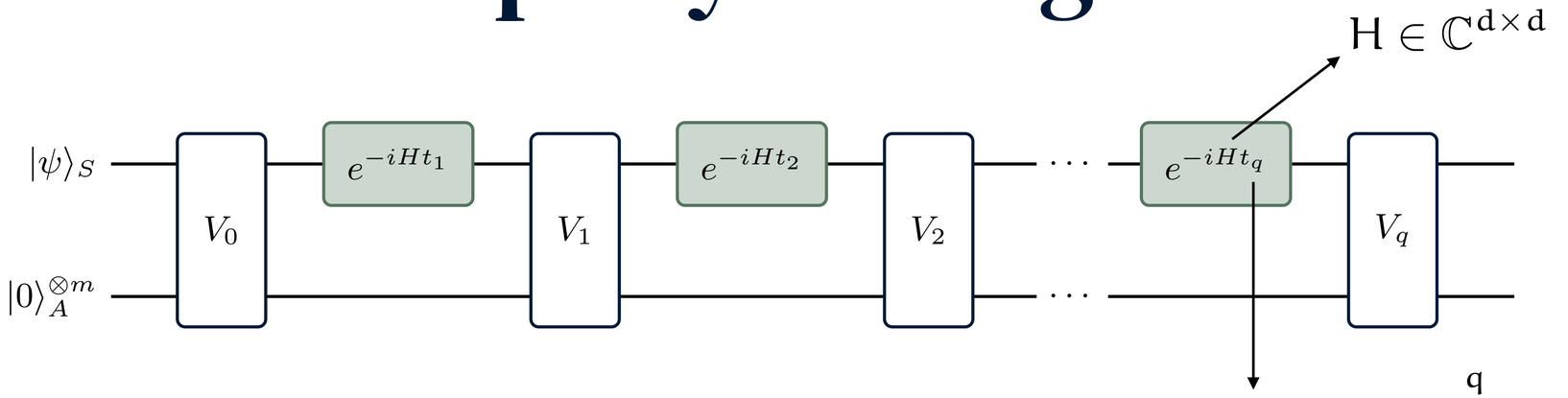
Hamiltonian Property Testing



Hamiltonian Property Testing



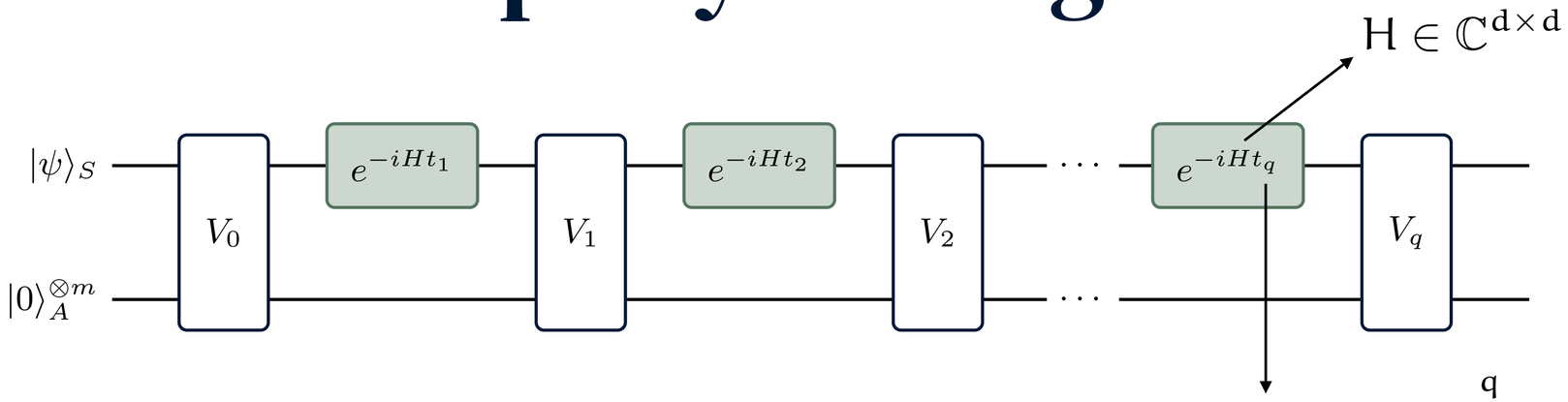
Hamiltonian Property Testing



Total evolution time: $T = \sum_{j=1}^{q} |t_j|$

Without time reversal $t_j > 0$; otherwise, $t_j \in \mathbb{R}$

Hamiltonian Property Testing



Without time reversal $t_j > 0$; otherwise, $t_j \in \mathbb{R}$

Problem (Hamiltonian certification [BCO'26]).

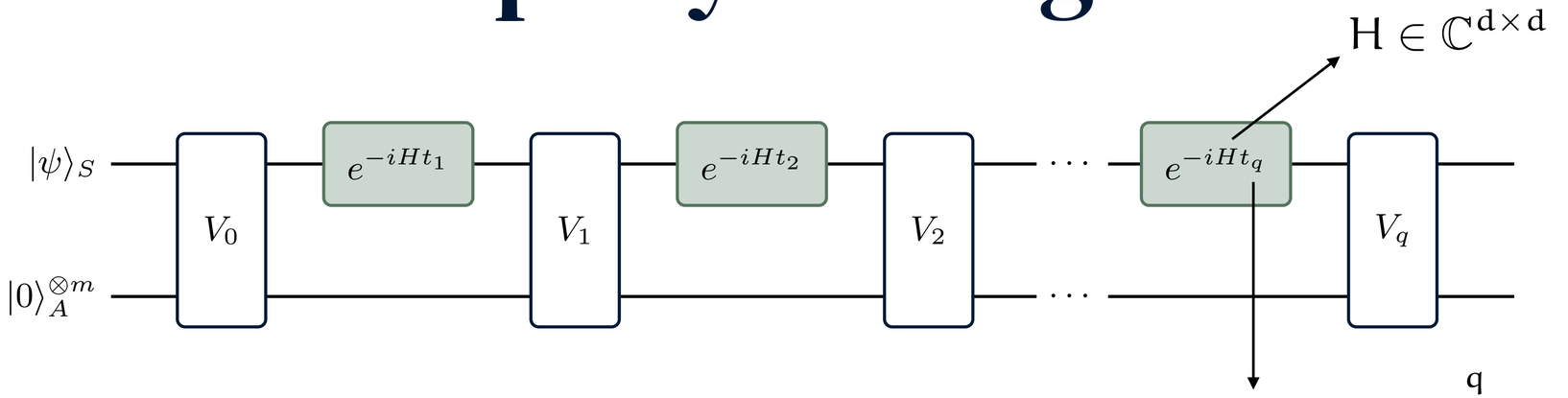
Let H_0 be a known target Hamiltonian. Decide:

(i) if $H = H_0$, accept w/prob. $\geqslant 2/3$

(ii) if $\frac{1}{\sqrt{d}}\|H - H_0\|_F > \varepsilon$, reject w/prob. $\geqslant 2/3$

[BCO'26] Bluhm, Caro, Oufkir. Hamiltonian property testing.

Hamiltonian Property Testing



Without time reversal $t_j > 0$; otherwise, $t_j \in \mathbb{R}$

Problem (Hamiltonian certification [BCO'26]).

Let H_0 be a known target Hamiltonian. Decide:

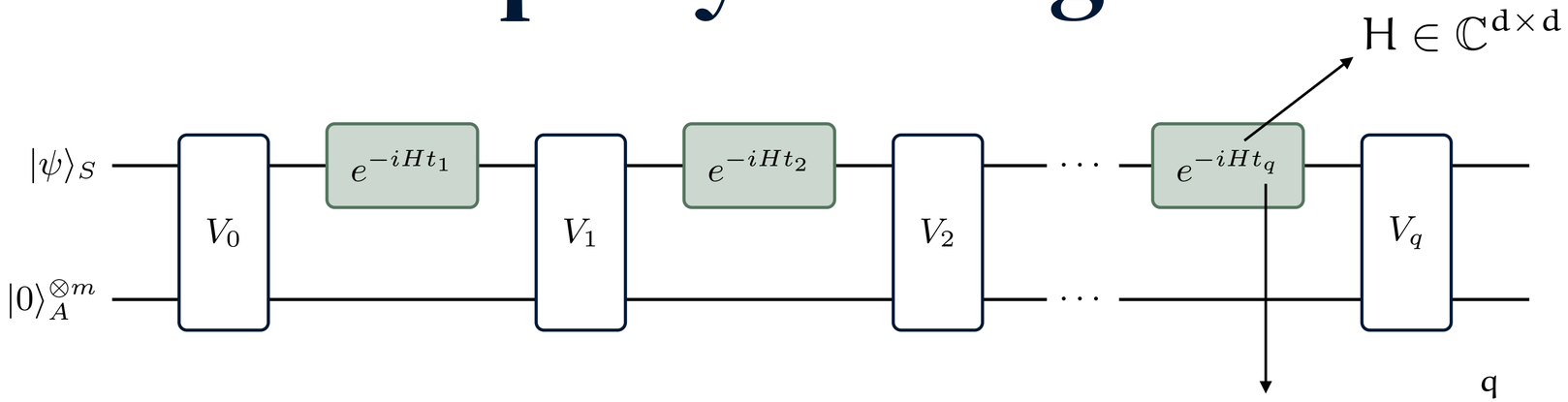
(i) if $H = H_0$, accept w/prob. $\geqslant 2/3$

(ii) if $\frac{1}{\sqrt{d}}\|H - H_0\|_F > \varepsilon$, reject w/prob. $\geqslant 2/3$

Average-case rate of distinguishability

[BCO'26] Bluhm, Caro, Oufkir. Hamiltonian property testing.

Hamiltonian Property Testing



Without time reversal $t_j > 0$; otherwise, $t_j \in \mathbb{R}$

Problem (Hamiltonian locality testing [BCO'26]).

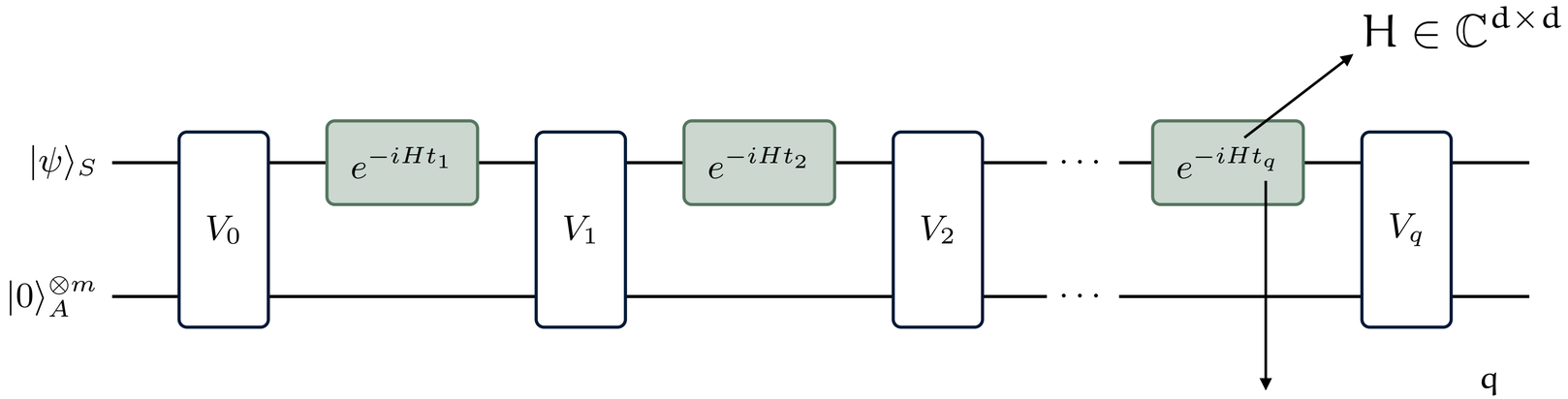
Let k be a positive integer. Decide:

(i) if H is k-local, accept w/prob. $\geqslant 2/3$

(ii) if H is ε-far from k-local, reject w/prob. $\geqslant 2/3$

[BCO'26] Bluhm, Caro, Oufkir. Hamiltonian property testing.

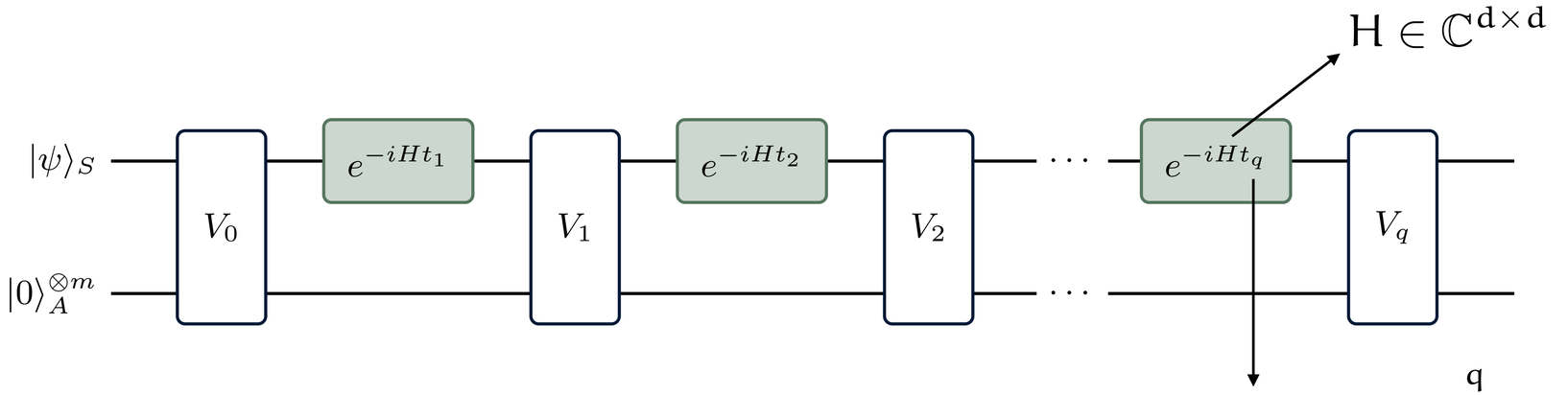
Prior Work



Total evolution time: $T = \sum_{j=1}^{q} |t_j|$

Without time reversal $t_j > 0$; otherwise, $t_j \in \mathbb{R}$

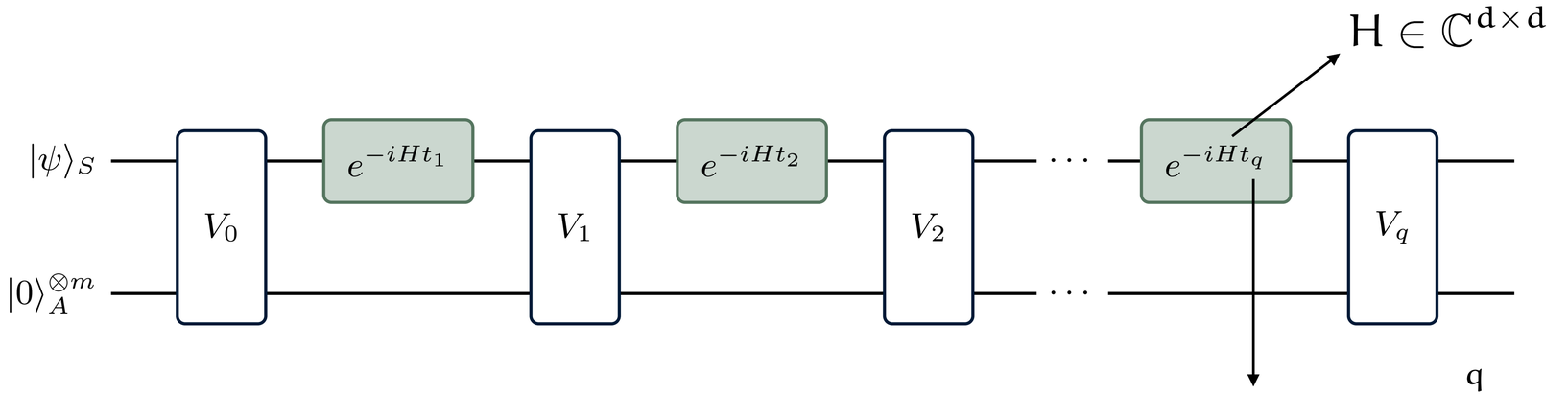
Prior Work



Without time reversal $t_j > 0$; otherwise, $t_j \in \mathbb{R}$

- With time reversal, $T = O(1/\varepsilon)$ suffices (*Heisenberg limit*) [KL'25,Gao+'26]

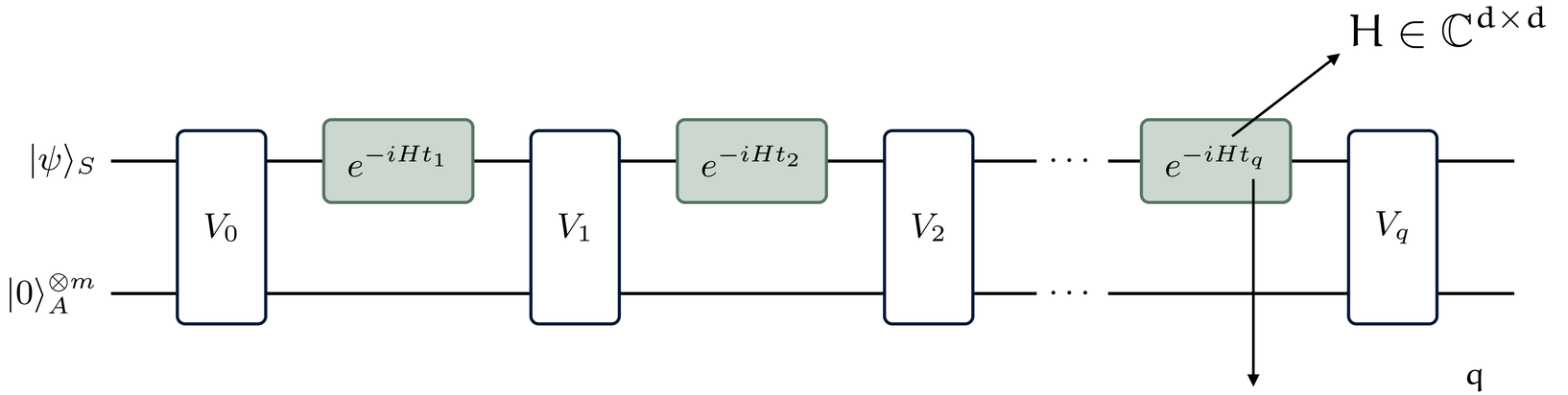
Prior Work



Without time reversal $t_j > 0$; otherwise, $t_j \in \mathbb{R}$

- With time reversal, $T = O(1/\varepsilon)$ suffices (*Heisenberg limit*) [KL'25,Gao+'26]
- Without time reversal, best known algorithms use $T = O(1/\varepsilon^2)$ (*standard quantum limit*) [KL'25,ST'25]

Prior Work



Without time reversal $t_j > 0$; otherwise, $t_j \in \mathbb{R}$

- With time reversal, $T = O(1/\varepsilon)$ suffices (*Heisenberg limit*) [KL'25,Gao+'26]
- Without time reversal, best known algorithms use $T = O(1/\varepsilon^2)$ (*standard quantum limit*) [KL'25,ST'25]

Q: is time reversal required for Heisenberg-limited scaling?

[KL'25] Kallaugher, Liang. Hamiltonian locality testing via Trotterized postselection.
[Gao+'26] Gao et al. Quantum Hamiltonian certification.
[ST'25] Sinha, Tong. Improved Hamiltonian Learning and Sparsity Testing through Bell Sampling.

Prior Work

Problem (Amplitude Estimation).

Given U on $n+1$ qubits such that

$$U|0^{n+1}\rangle = a|\phi_1\rangle|1\rangle + \sqrt{1-a^2}|\phi_0\rangle|0\rangle$$

output $\hat{a}$ such that $|\hat{a}-a| < \varepsilon$ w/prob. $\geqslant 2/3$, using as few queries to U and $U^\dagger$ as possible.

Prior Work

Problem (Amplitude Estimation).

Given U on $n+1$ qubits such that

$$U|0^{n+1}\rangle = a|\phi_1\rangle|1\rangle + \sqrt{1-a^2}|\phi_0\rangle|0\rangle$$

output $\hat{a}$ such that $|\hat{a}-a| < \varepsilon$ w/prob. $\geqslant 2/3$, using as few queries to U and $U^\dagger$ as possible.

- Can solve using $O(1/\varepsilon)$ queries to U and $U^\dagger$ [BHMT'98]
- Without inverse access, $\Omega(1/\varepsilon^2)$ queries are required whenever $\varepsilon \gg 1/\sqrt{d}$ [TW'26]

Prior Work

Problem (Amplitude Estimation).

Given U on $n+1$ qubits such that

$$U|0^{n+1}\rangle = a|\phi_1\rangle|1\rangle + \sqrt{1-a^2}|\phi_0\rangle|0\rangle$$

output $\hat{a}$ such that $|\hat{a}-a| < \varepsilon$ w/prob. $\geqslant 2/3$, using as few queries to U and $U^\dagger$ as possible.

- Can solve using $O(1/\varepsilon)$ queries to U and $U^\dagger$ [BHMT'98]
- Without inverse access, $\Omega(1/\varepsilon^2)$ queries are required whenever $\varepsilon \gg 1/\sqrt{d}$ [TW'26]

$\varepsilon \ll 1/\sqrt{d}$ is the "ultra-high precision regime"

Prior Work

Problem (Amplitude Estimation).

Given U on $n+1$ qubits such that

$$U|0^{n+1}\rangle = a|\phi_1\rangle|1\rangle + \sqrt{1-a^2}|\phi_0\rangle|0\rangle$$

output $\hat{a}$ such that $|\hat{a}-a| < \varepsilon$ w/prob. $\geqslant 2/3$, using as few queries to U and $U^\dagger$ as possible.

- Can solve using $O(1/\varepsilon)$ queries to U and $U^\dagger$ [BHMT'98]
- Without inverse access, $\Omega(1/\varepsilon^2)$ queries are required whenever $\varepsilon \gg 1/\sqrt{d}$ [TW'26]

$\varepsilon \ll 1/\sqrt{d}$ is the "ultra-high precision regime"

Q: Does this prove time reversal is required?

Prior Work

Problem (Amplitude Estimation).

Given U on $n+1$ qubits such that

$$U|0^{n+1}\rangle = a|\phi_1\rangle|1\rangle + \sqrt{1-a^2}|\phi_0\rangle|0\rangle$$

output $\hat{a}$ such that $|\hat{a}-a| < \varepsilon$ w/prob. $\geqslant 2/3$, using as few queries to U and $U^\dagger$ as possible.

- Can solve using $O(1/\varepsilon)$ queries to U and $U^\dagger$ [BHMT'98]
- Without inverse access, $\Omega(1/\varepsilon^2)$ queries are required whenever $\varepsilon \gg 1/\sqrt{d}$ [TW'26]

$\varepsilon \ll 1/\sqrt{d}$ is the "ultra-high precision regime"

Q: Does this prove time reversal is required?

A: No, discrete queries are different. ([HTFS'23] vs [DOS'24].)

[BBHT'98] Brassard et al. Quantum Amplitude amplification and estimation.
[TW'26] Tang, Wright. Amplitude amplification and estimation require inverses.
[HTFS'23] Huang et al. Learning many-body Hamiltonians with Heisenberg-limited scaling.
[DOS'24] Dutkiewicz, O'Brien, Schuster. The advantage of quantum control in many-body Hamiltonian learning.

Discrete Queries are Different

1. Intuitively, continuous is different because of new tools, e.g., Trotterization.
2. In Hamiltonian learning, $O(1/\varepsilon)$ evolution time is achievable with continuous queries, but with discrete queries (long time) you are restricted to $\Omega(1/\varepsilon^2)$.
3. Compressed oracle (as in TW proof) breaks in continuous time.

Discrete Queries are Different

1. Intuitively, continuous is different because of new tools, e.g., Trotterization.
2. In Hamiltonian learning, $O(1/\varepsilon)$ evolution time is achievable with continuous queries, but with discrete queries (long time) you are restricted to $\Omega(1/\varepsilon^2)$.
3. Compressed oracle (as in TW proof) breaks in continuous time.

Discrete Queries are Different

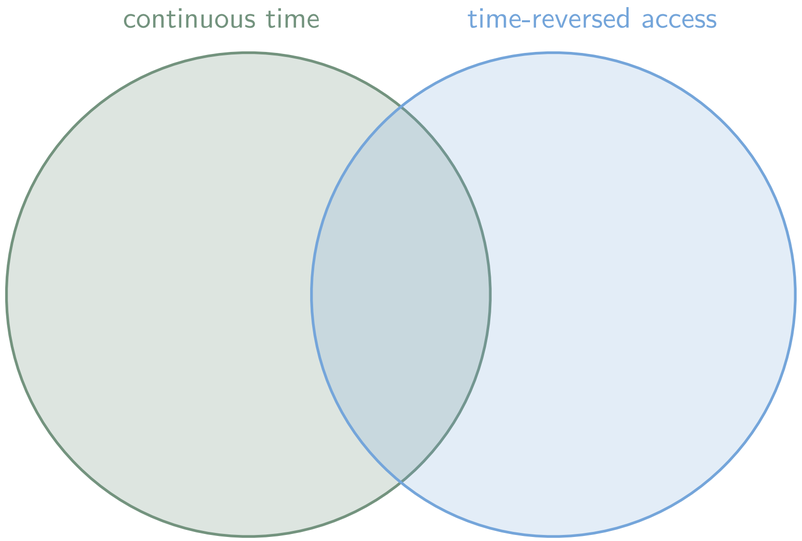
1. Intuitively, continuous is different because of new tools, e.g., Trotterization.
2. In Hamiltonian learning, $O(1/\varepsilon)$ evolution time is achievable with continuous queries, but with discrete queries (long time) you are restricted to $\Omega(1/\varepsilon^2)$.
3. Compressed oracle (as in TW proof) breaks in continuous time.

Key Technical Challenge

- We need a quantum lower bound technique that:
  (i) works in the continuous-time setting
  (ii) Is sensitive to forward-only versus time-reversed access

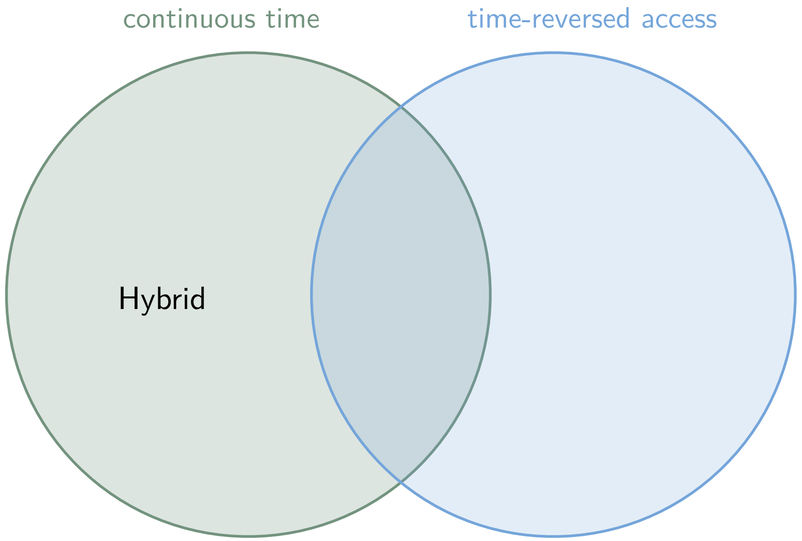
Key Technical Challenge

- We need a quantum lower bound technique that:
  (i) works in the continuous-time setting
  (ii) Is sensitive to forward-only versus time-reversed access



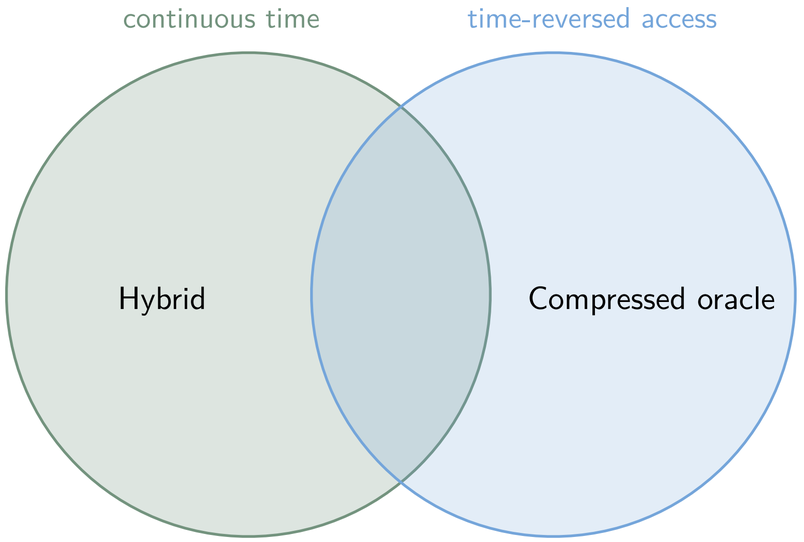
Key Technical Challenge

- We need a quantum lower bound technique that:
  (i) works in the continuous-time setting
  (ii) Is sensitive to forward-only versus time-reversed access



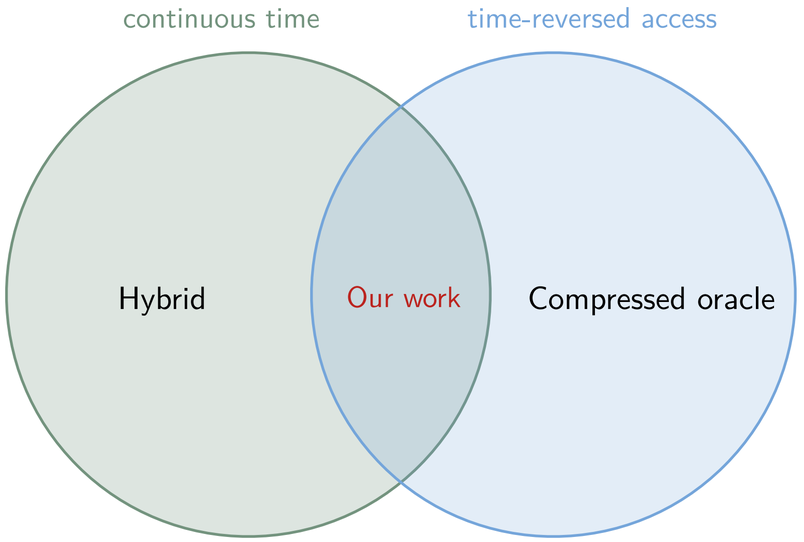
Key Technical Challenge

- We need a quantum lower bound technique that:
  (i) works in the continuous-time setting
  (ii) Is sensitive to forward-only versus time-reversed access



Key Technical Challenge

- We need a quantum lower bound technique that:
  (i) works in the continuous-time setting
  (ii) Is sensitive to forward-only versus time-reversed access



Overview

1. Background & Problem Statement
2. Main Results
3. Proof Sketch
4. Conclusion & Future Directions

Hamiltonian Certification

Theorem (Hamiltonian certification without time reversal [This work]).

Any quantum algorithm without time reversal for Hamiltonian certification requires a total evolution time

$$T = \Omega\left(\min\left\{\frac{1}{\varepsilon^2}, \frac{\sqrt{d}}{\varepsilon}\right\}\right).$$

Hamiltonian Certification

Theorem (Hamiltonian certification without time reversal [This work]).

Any quantum algorithm without time reversal for Hamiltonian certification requires a total evolution time

$$T = \Omega\left(\min\left\{\frac{1}{\varepsilon^2}, \frac{\sqrt{d}}{\varepsilon}\right\}\right).$$

*See also [GLZ'26], which independently established $\Omega(1/\varepsilon^2)$ using different techniques

[GLZ'26] Gutiérrez, Lee, Zur. Hamiltonian locality testing and certification do not achieve the Heisenberg limit

Hamiltonian Certification

Theorem (Hamiltonian certification without time reversal [This work]).

Any quantum algorithm without time reversal for Hamiltonian certification requires a total evolution time

$$T = \Omega\left(\min\left\{\frac{1}{\varepsilon^2}, \frac{\sqrt{d}}{\varepsilon}\right\}\right).$$

*See also [GLZ'26], which independently established $\Omega(1/\varepsilon^2)$ using different techniques

- In fact, this is tight

[GLZ'26] Gutiérrez, Lee, Zur. Hamiltonian locality testing and certification do not achieve the Heisenberg limit

Hamiltonian Certification

Theorem (Hamiltonian certification without time reversal [This work]).

Any quantum algorithm without time reversal for Hamiltonian certification requires a total evolution time

$$T = \Omega\left(\min\left\{\frac{1}{\varepsilon^2}, \frac{\sqrt{d}}{\varepsilon}\right\}\right).$$

*See also [GLZ'26], which independently established $\Omega(1/\varepsilon^2)$ using different techniques

- In fact, this is tight
- And for certification with arbitrary Schatten p-norm (from average- to worst-case distinguishability):

$$T = \Theta\left(\min\left\{\frac{1}{\varepsilon^p}, \frac{d^{1-1/p}}{\varepsilon}\right\}\right)$$

[GLZ'26] Gutiérrez, Lee, Zur. Hamiltonian locality testing and certification do not achieve the Heisenberg limit

Hamiltonian Certification

Theorem (Hamiltonian certification without time reversal [This work]).

Any quantum algorithm without time reversal for Hamiltonian certification requires a total evolution time

$$T = \Omega\left(\min\left\{\frac{1}{\varepsilon^2}, \frac{\sqrt{d}}{\varepsilon}\right\}\right).$$

*See also [GLZ'26], which independently established $\Omega(1/\varepsilon^2)$ using different techniques

- In fact, this is tight
- And for certification with arbitrary Schatten p-norm (from average- to worst-case distinguishability):

$$T = \Theta\left(\min\left\{\frac{1}{\varepsilon^p}, \frac{d^{1-1/p}}{\varepsilon}\right\}\right)$$

- Same lower bound holds for locality testing

[GLZ'26] Gutiérrez, Lee, Zur. Hamiltonian locality testing and certification do not achieve the Heisenberg limit

Amplitude Estimation

Corollary (Amplitude estimation without inverses [This work]).

Given only the ability to apply U and not its inverse, any algorithm for amplitude estimation requires

$$\Omega\left(\min\left\{\frac{1}{\varepsilon^2}, \frac{\sqrt{d}}{\varepsilon}\right\}\right)$$

queries, even with fractional queries.

*See also [GLZ'26], which independently established $\Omega(1/\varepsilon^2)$ using different techniques

[GLZ'26] Gutiérrez, Lee, Zur. Hamiltonian locality testing and certification do not achieve the Heisenberg limit

[TW'26] Tang, Wright. Amplitude amplification and estimation require inverses.

Amplitude Estimation

Corollary (Amplitude estimation without inverses [This work]).

Given only the ability to apply U and not its inverse, any algorithm for amplitude estimation requires

$$\Omega\left(\min\left\{\frac{1}{\varepsilon^2}, \frac{\sqrt{d}}{\varepsilon}\right\}\right)$$

queries, even with fractional queries.

*See also [GLZ'26], which independently established $\Omega(1/\varepsilon^2)$ using different techniques

- Extends results in [TW'26] to fractional queries

[GLZ'26] Gutiérrez, Lee, Zur. Hamiltonian locality testing and certification do not achieve the Heisenberg limit
[TW'26] Tang, Wright. Amplitude amplification and estimation require inverses.

Amplitude Estimation

Corollary (Amplitude estimation without inverses [This work]).

Given only the ability to apply U and not its inverse, any algorithm for amplitude estimation requires

$$\Omega\left(\min\left\{\frac{1}{\varepsilon^2}, \frac{\sqrt{\mathrm{d}}}{\varepsilon}\right\}\right)$$

queries, even with fractional queries.

*See also [GLZ'26], which independently established $\Omega(1/\varepsilon^2)$ using different techniques

- Extends results in [TW'26] to fractional queries
- Improved lower bound in the high-precision regime where $\varepsilon \ll \mathrm{d}^{-1/2}$

[GLZ'26] Gutiérrez, Lee, Zur. Hamiltonian locality testing and certification do not achieve the Heisenberg limit
[TW'26] Tang, Wright. Amplitude amplification and estimation require inverses.

No Robust Quantum Advantage in Search

Let

$$O_f^\theta = \sum_{x=1}^{d} e^{i\theta_x f(x)} |x\rangle\langle x|$$

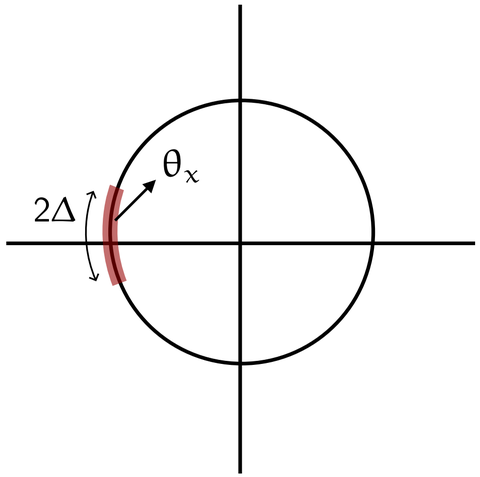
for some unknown angles $\theta_x \in [\pi - \Delta, \pi + \Delta]$ and unknown f that marks $\Theta(1)$ elements.

No Robust Quantum Advantage in Search

Let

$$O_f^\theta = \sum_{x=1}^{d} e^{i\theta_x f(x)} |x\rangle\langle x|$$

for some unknown angles $\theta_x \in [\pi - \Delta, \pi + \Delta]$ and unknown f that marks $\Theta(1)$ elements.

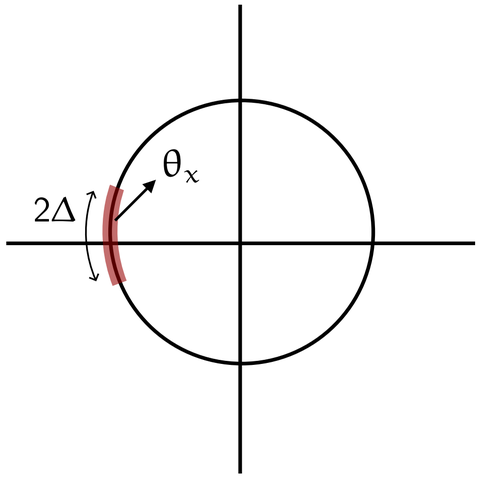


No Robust Quantum Advantage in Search

Let

$$O_f^\theta = \sum_{x=1}^{d} e^{i\theta_x f(x)} |x\rangle\langle x|$$

for some unknown angles $\theta_x \in [\pi - \Delta, \pi + \Delta]$ and unknown f that marks $\Theta(1)$ elements.



Theorem (No-go for quantum search [This work]).

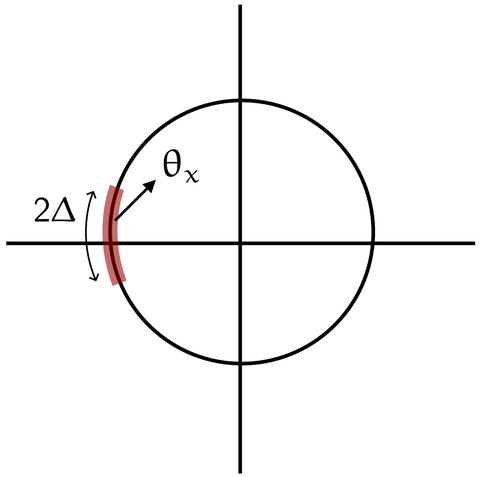
Given only the ability to apply O_f^θ and not its inverse, any algorithm for search requires at least $\Omega(d\Delta)$ applications of O_f^θ.

No Robust Quantum Advantage in Search

Let

$$O_f^\theta = \sum_{x=1}^{d} e^{i\theta_x f(x)} |x\rangle\langle x|$$

for some unknown angles $\theta_x \in [\pi - \Delta, \pi + \Delta]$ and unknown f that marks $\Theta(1)$ elements.



Theorem (No-go for quantum search [This work]).

Given only the ability to apply O_f^θ and not its inverse, any algorithm for search requires at least $\Omega(d\Delta)$ applications of O_f^θ.

- By contrast, with inverse access, $O(\sqrt{d})$ applications suffice [HMdW'03]

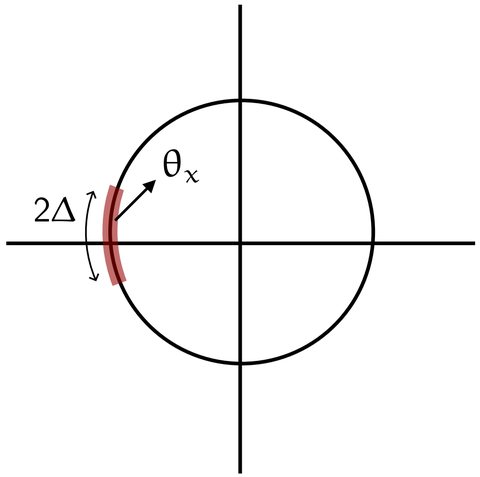
[HMdW'03] Hoyer, Mosca, de Wolfe. Quantum search with bounded-error inputs.
[RS'12] Regev, Schiff. Impossibility of a Quantum Speed-up with a Faulty Oracle.

No Robust Quantum Advantage in Search

Let

$$O_f^\theta = \sum_{x=1}^{d} e^{i\theta_x f(x)} |x\rangle\langle x|$$

for some unknown angles $\theta_x \in [\pi - \Delta, \pi + \Delta]$ and unknown f that marks $\Theta(1)$ elements.



Theorem (No-go for quantum search [This work]).

Given only the ability to apply O_f^θ and not its inverse, any algorithm for search requires at least $\Omega(d\Delta)$ applications of O_f^θ.

- By contrast, with inverse access, $O(\sqrt{d})$ applications suffice [HMdW'03]
- Distinct from no-go results based on noisy channels [RS'12]

[HMdW'03] Hoyer, Mosca, de Wolfe. Quantum search with bounded-error inputs.
[RS'12] Regev, Schiff. Impossibility of a Quantum Speed-up with a Faulty Oracle.

Overview

1. Background & Problem Statement
2. Main Results
3. Proof Sketch
4. Conclusion & Future Directions

Proof Checklist

1. Construct the hardest possible random Hamiltonian H to distinguish from zero
2. Check that it is still ε-far, so the certification algorithm is successful
3. Bound distinguishability as a function of total evolution time

New techniques needed: (i) hybrid method and (ii) compressed oracle method fail

A Hard Distinguishing Task

$$H = \sum_{j=1}^{B} h_j \Pi_j = \begin{pmatrix} h_1 I_r & 0 & \cdots & 0 \\ 0 & h_2 I_r & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & h_B I_r \end{pmatrix}$$

A Hard Distinguishing Task

$$H = \sum_{j=1}^{B} h_j \Pi_j = \begin{pmatrix} h_1 I_r & 0 & \cdots & 0 \\ 0 & h_2 I_r & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & h_B I_r \end{pmatrix}$$

where $d = rB$ and $h_1, \ldots, h_B$ are iid copies of $\beta\theta$, for $\beta \sim \mathrm{Ber}(p)$ and $\theta \sim \mathrm{Unif}([-\sigma, \sigma])$

A Hard Distinguishing Task

$$H = \sum_{j=1}^{B} h_j \Pi_j = \begin{pmatrix} h_1 I_r & 0 & \cdots & 0 \\ 0 & h_2 I_r & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & h_B I_r \end{pmatrix}$$

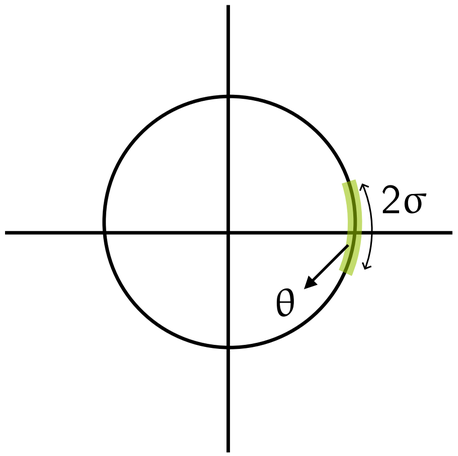
Mostly zero

where $d = rB$ and $h_1, \ldots, h_B$ are iid copies of $\beta\theta$, for $\beta \sim \mathrm{Ber}(p)$ and $\theta \sim \mathrm{Unif}([-\sigma, \sigma])$

A Hard Distinguishing Task

$$H = \sum_{j=1}^{B} h_j \Pi_j = \begin{pmatrix} h_1 I_r & 0 & \cdots & 0 \\ 0 & h_2 I_r & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & h_B I_r \end{pmatrix}$$

Mostly zero

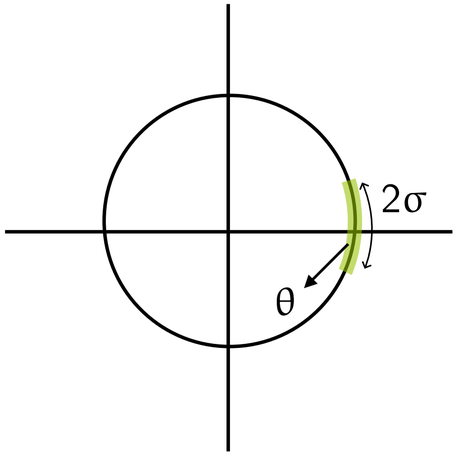


where $d = rB$ and $h_1, \ldots, h_B$ are iid copies of $\beta\theta$, for $\beta \sim \mathrm{Ber}(p)$ and $\theta \sim \mathrm{Unif}([-\sigma, \sigma])$

A Hard Distinguishing Task

$$H = \sum_{j=1}^{B} h_j \Pi_j = \begin{pmatrix} h_1 I_r & 0 & \cdots & 0 \\ 0 & h_2 I_r & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & h_B I_r \end{pmatrix}$$

Mostly zero



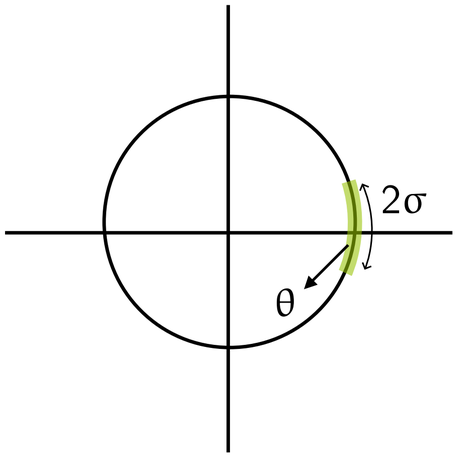
where $d = rB$ and $h_1, \ldots, h_B$ are iid copies of $\beta\theta$, for $\beta \sim \mathrm{Ber}(p)$ and $\theta \sim \mathrm{Unif}([-\sigma, \sigma])$

Q1: is H ε-far from zero?

A Hard Distinguishing Task

$$H = \sum_{j=1}^{B} h_j \Pi_j = \begin{pmatrix} h_1 I_r & 0 & \cdots & 0 \\ 0 & h_2 I_r & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & h_B I_r \end{pmatrix}$$

Mostly zero



where $d = rB$ and $h_1, \ldots, h_B$ are iid copies of $\beta\theta$, for $\beta \sim \mathrm{Ber}(p)$ and $\theta \sim \mathrm{Unif}([-\sigma, \sigma])$

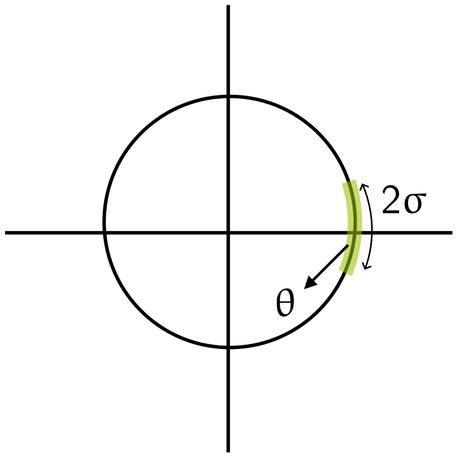
Q1: is H ε-far from zero?

A: if, $p \sim 1/B$, $B \sim 1/\varepsilon^2$, $\sigma = \Theta(1)$ then yes.

A Hard Distinguishing Task

$$H = \sum_{j=1}^{B} h_j \Pi_j = \begin{pmatrix} h_1 I_r & 0 & \cdots & 0 \\ 0 & h_2 I_r & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & h_B I_r \end{pmatrix}$$

Mostly zero



where $d = rB$ and $h_1, \ldots, h_B$ are iid copies of $\beta\theta$, for $\beta \sim \mathrm{Ber}(p)$ and $\theta \sim \mathrm{Unif}([-\sigma, \sigma])$

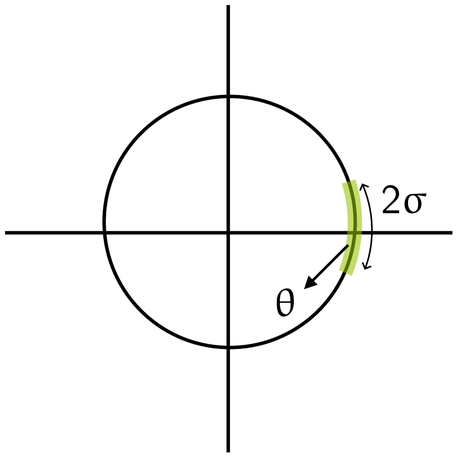
Q1: is H ε-far from zero?
A: if, $p \sim 1/B$, $B \sim 1/\varepsilon^2$, $\sigma = \Theta(1)$ then yes.

Q2: is H hard to distinguish from zero?

A Hard Distinguishing Task

$$H = \sum_{j=1}^{B} h_j \Pi_j = \begin{pmatrix} h_1 I_r & 0 & \cdots & 0 \\ 0 & h_2 I_r & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & h_B I_r \end{pmatrix}$$

Mostly zero



where $d = rB$ and $h_1, \ldots, h_B$ are iid copies of $\beta\theta$, for $\beta \sim \mathrm{Ber}(p)$ and $\theta \sim \mathrm{Unif}([-\sigma, \sigma])$

Q1: is H ε-far from zero?

A: if, $p \sim 1/B$, $B \sim 1/\varepsilon^2$, $\sigma = \Theta(1)$ then yes.

Theorem.

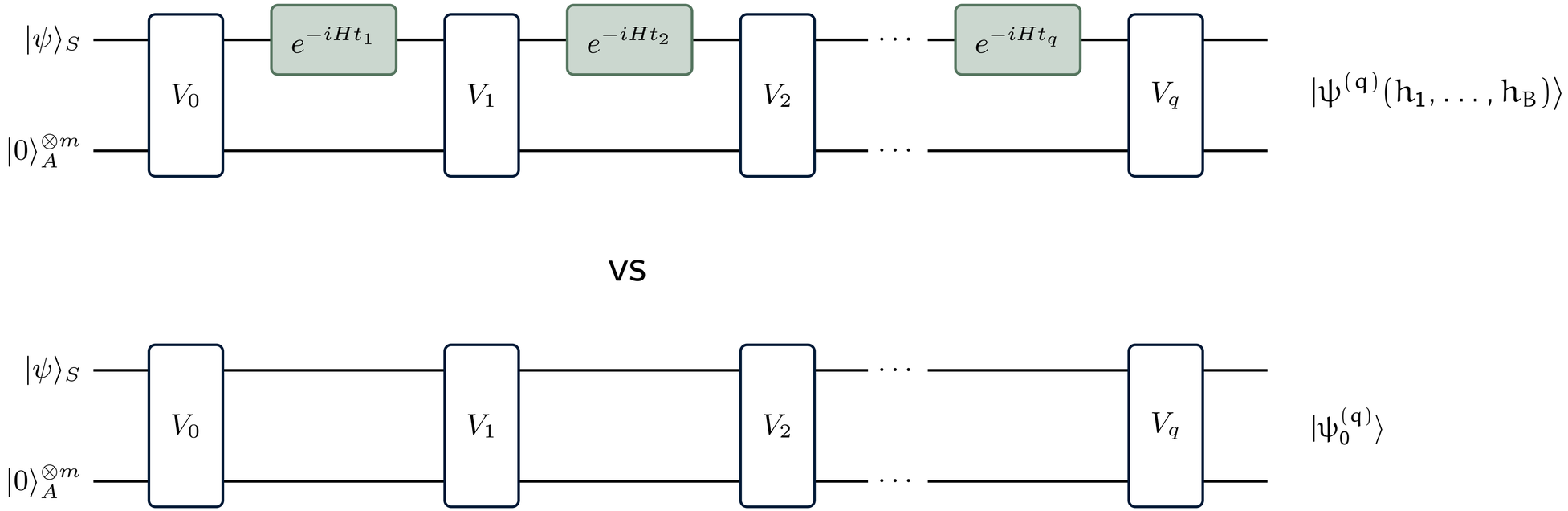
Any quantum algorithm to distinguish H from zero without time reversal requires a total evolution time $T = \Omega(1/\varepsilon^2)$.

Proof Checklist

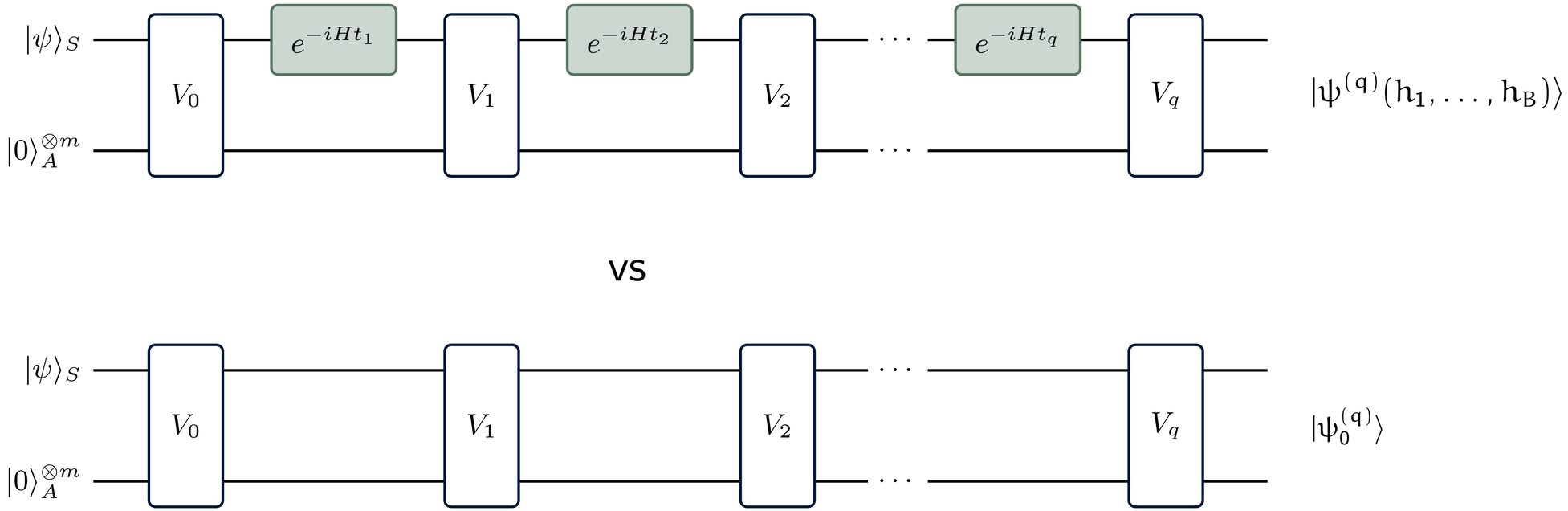
1. ~~Construct the hardest possible random Hamiltonian H to distinguish from zero~~
2. ~~Check that it is still ϵ far, so the certification algorithm is successful~~
3. Bound distinguishability as a function of total evolution time

New techniques needed: (i) hybrid method and (ii) compressed oracle method fail

Bounding distinguishability

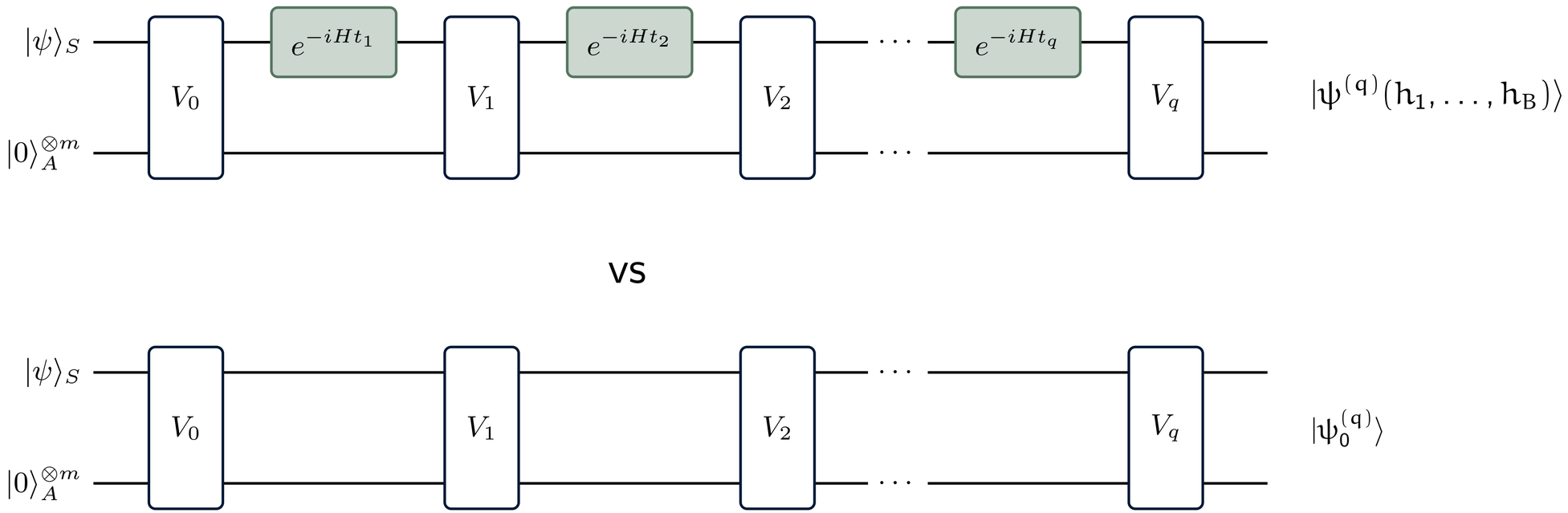


Bounding distinguishability



Squared trace distance upper bounded by $\mathfrak{p} \coloneqq \mathbb{E}\,\|(I - |\psi_0^{(q)}\rangle\langle\psi_0^{(q)}|)|\psi^{(q)}(h_1,\ldots,h_B)\rangle\|_2^2$

Bounding distinguishability

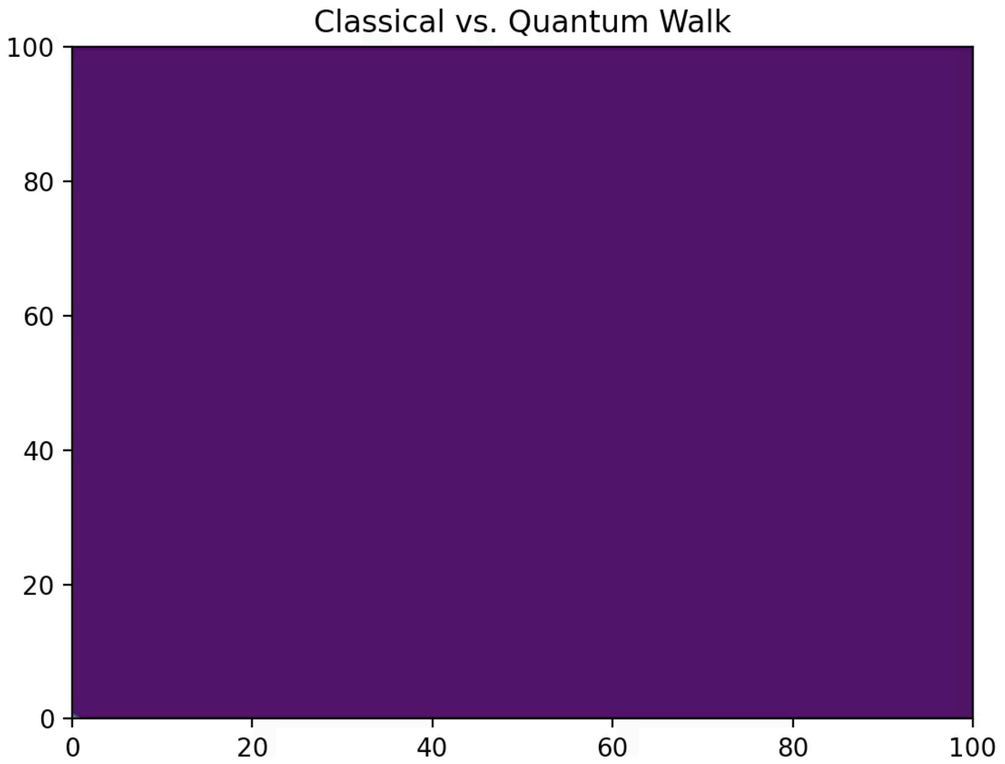


Squared trace distance upper bounded by $\mathfrak{p} \coloneqq \mathbb{E}\,\|(I - |\psi_0^{(q)}\rangle\langle\psi_0^{(q)}|)|\psi^{(q)}(h_1,\ldots,h_B)\rangle\|_2^2$

Q: Can we show $\mathfrak{p} \leqslant O(\varepsilon^2 T)$?

Note: *not* $O(\varepsilon^2 T^2)$

Diffusive vs ballistic growth of *distinguishability*


Classical vs. Quantum Walk
100
80
60
40
20
0
0
20
40
60
80
100

First Attempt: Hybrid Method

$$\mathfrak{p} \coloneqq \mathbb{E}\,\|(I - |\psi_0^{(q)}\rangle\langle\psi_0^{(q)}|)|\psi^{(q)}(h_1,\ldots,h_B)\rangle\|_2^2 \qquad \textbf{Want: } \mathfrak{p} \leqslant O(\varepsilon^2 T)$$

- Telescope over queries $j = 1,\ldots,q$ to get

First Attempt: Hybrid Method

$$\mathfrak{p} \coloneqq \mathbb{E}\,\|(I - |\psi_0^{(q)}\rangle\langle\psi_0^{(q)}|)|\psi^{(q)}(h_1,\ldots,h_B)\rangle\|_2^2 \qquad \textbf{Want: } \mathfrak{p} \leqslant O(\varepsilon^2 T)$$

- Telescope over queries $j = 1,\ldots,q$ to get

$$\begin{aligned}\||\psi^{(q)}(h_1,\ldots,h_B)\rangle - |\psi_0^{(q)}\rangle\| &\leqslant \sum_{j=1}^{q} \left\| V_q e^{-iHt_q} \cdots V_j (e^{-iHt_j} - I) V_{j-1} \cdots V_0 |\psi_{in}\rangle \right\| \\ &= \sum_{j=1}^{q} \left\| (e^{-iHt_j} - I) V_{j-1} \cdots V_0 |\psi_{in}\rangle \right\|\end{aligned}$$

First Attempt: Hybrid Method

$$\mathfrak{p} \coloneqq \mathbb{E}\,\|(I - |\psi_0^{(q)}\rangle\langle\psi_0^{(q)}|)|\psi^{(q)}(h_1,\ldots,h_B)\rangle\|_2^2 \qquad \textbf{Want: } \mathfrak{p} \leqslant O(\varepsilon^2 T)$$

- Telescope over queries $j = 1,\ldots,q$ to get

$$\begin{aligned}\||\psi^{(q)}(h_1,\ldots,h_B)\rangle - |\psi_0^{(q)}\rangle\| &\leqslant \sum_{j=1}^{q} \left\| V_q e^{-iHt_q}\cdots V_j(e^{-iHt_j} - I)V_{j-1}\cdots V_0|\psi_{in}\rangle \right\| \\ &= \sum_{j=1}^{q} \left\| (e^{-iHt_j} - I)V_{j-1}\cdots V_0|\psi_{in}\rangle \right\|\end{aligned}$$

- **Problem:** terms are invariant under $t_j \mapsto -t_j \implies$ cannot possibly be sensitive to time reversal!

First Attempt: Hybrid Method

$$\mathfrak{p} \coloneqq \mathbb{E}\,\|(I - |\psi_0^{(q)}\rangle\langle\psi_0^{(q)}|)|\psi^{(q)}(h_1,\ldots,h_B)\rangle\|_2^2 \qquad \textbf{Want: } \mathfrak{p} \leqslant O(\varepsilon^2 T)$$

- Telescope over queries $j = 1,\ldots,q$ to get

$$\begin{aligned}\||\psi^{(q)}(h_1,\ldots,h_B)\rangle - |\psi_0^{(q)}\rangle\| &\leqslant \sum_{j=1}^{q} \left\|V_q e^{-iHt_q}\cdots V_j(e^{-iHt_j} - I)V_{j-1}\cdots V_0|\psi_{in}\rangle\right\| \\ &= \sum_{j=1}^{q} \left\|(e^{-iHt_j} - I)V_{j-1}\cdots V_0|\psi_{in}\rangle\right\|\end{aligned}$$

- **Problem:** terms are invariant under $t_j \mapsto -t_j \implies$ cannot possibly be sensitive to time reversal!
- Indeed, leads to ballistic growth upper bound, $\mathfrak{p} \leqslant O(\varepsilon^2 T^2)$

Isolate Hamiltonian Terms

$$\mathfrak{p} \coloneqq \mathbb{E}\,\|(I - |\psi_0^{(q)}\rangle\langle\psi_0^{(q)}|)|\psi^{(q)}(h_1, \ldots, h_B)\rangle\|_2^2 \qquad \textbf{Want: } \mathfrak{p} \leqslant O(\varepsilon^2 T)$$

- **Idea:** let's focus on *one random variable at a time*:

Isolate Hamiltonian Terms

$$\mathfrak{p} \coloneqq \mathbb{E} \left\| (I - |\psi_0^{(q)}\rangle\langle\psi_0^{(q)}|)|\psi^{(q)}(h_1, \ldots, h_B)\rangle \right\|_2^2 \qquad \textbf{Want: } \mathfrak{p} \leqslant O(\varepsilon^2 T)$$

- **Idea:** let's focus on *one random variable at a time*:
- *Efron-Stein inequality*: for iid random variables $x_1, \ldots, x_n$

$$\mathrm{Var}[f(x_1, \ldots, x_n)] \leqslant \frac{1}{2} \sum_{i=1}^{n} \mathbb{E}\left[\left(f(x_1, \ldots, x_i, \ldots, x_n) - f(x_1, \ldots, x_i', \ldots, x_n) \right)^2 \right]$$

where x_i' is a fresh copy of x_i

Isolate Hamiltonian Terms

$$\mathfrak{p} \coloneqq \mathbb{E}\,\|(I - |\psi_0^{(q)}\rangle\langle\psi_0^{(q)}|)|\psi^{(q)}(h_1,\ldots,h_B)\rangle\|_2^2 \qquad \textbf{Want: } \mathfrak{p} \leqslant O(\varepsilon^2 T)$$

- **Idea:** let's focus on *one random variable at a time*:
- *Efron-Stein inequality*: for iid random variables $x_1, \ldots, x_n$

$$\mathrm{Var}[f(x_1,\ldots,x_n)] \leqslant \frac{1}{2}\sum_{i=1}^{n} \mathbb{E}\left[\left(f(x_1,\ldots,x_i,\ldots,x_n) - f(x_1,\ldots,x_i',\ldots,x_n)\right)^2\right]$$

  where x_i' is a fresh copy of x_i

- Similar reasoning leads to:

$$\mathfrak{p} \leqslant \sum_{i=1}^{B} \||\Delta_i^{(q)}\rangle\|_2^2, \quad |\Delta_i^{(q)}\rangle \coloneqq |\psi^{(q)}(h_1,\ldots,h_i,\ldots,h_m)\rangle - |\psi^{(q)}(h_1,\ldots,0,\ldots,h_m)\rangle$$

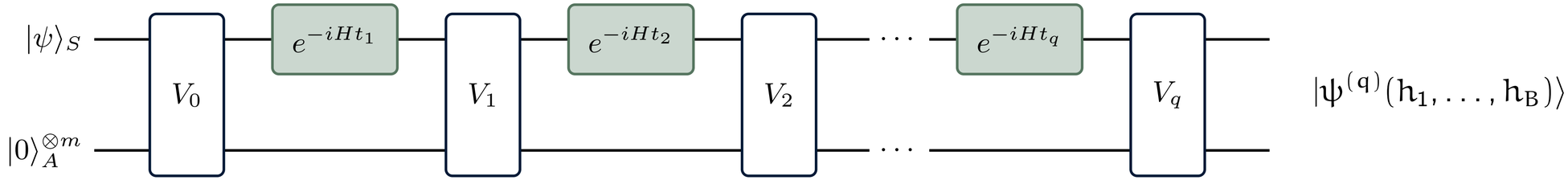
Diffusive Growth of Distinguishability

- Now we can focus on one subspace, say, Π_1

Diffusive Growth of Distinguishability

- Now we can focus on one subspace, say, Π_1
- Recall

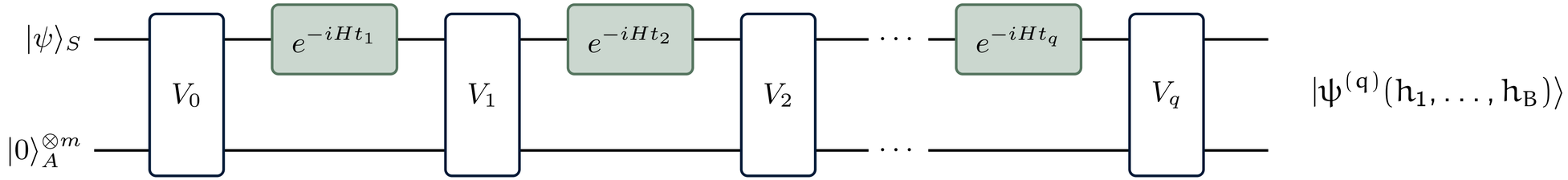
$$H = \sum_{j=1}^{B} h_j \Pi_j = \begin{pmatrix} h_1 I_r & 0 & \cdots & 0 \\ 0 & h_2 I_r & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & h_B I_r \end{pmatrix}$$



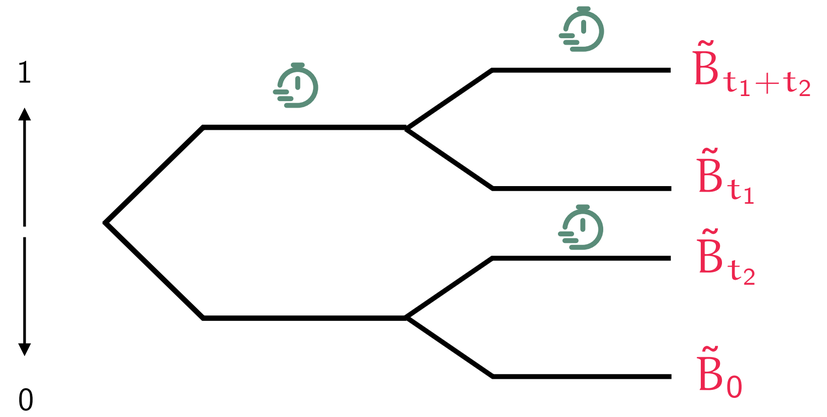
Diffusive Growth of Distinguishability

- Now we can focus on one subspace, say, Π_1
- Recall

$$H = \sum_{j=1}^{B} h_j \Pi_j = \begin{pmatrix} h_1 I_r & 0 & \cdots & 0 \\ 0 & h_2 I_r & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & h_B I_r \end{pmatrix}$$

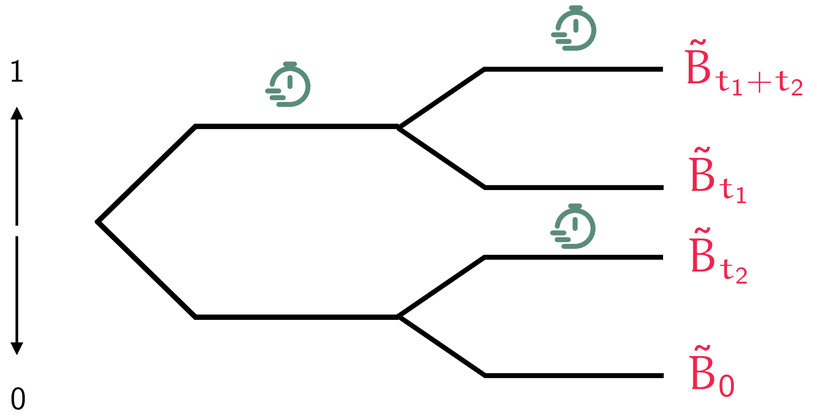


$$\begin{aligned} |\Delta_1^{(q)}\rangle &:= |\psi^{(q)}(h_1, \ldots, h_m)\rangle - |\psi^{(q)}(0, \ldots, h_m)\rangle \\ &= \sum_{z \in \{0,1\}^q} (e^{-ih_1 \sum_{j:z_j=1} t_j} - 1)|\varphi_z\rangle \\ &=: \sum_{z \in \{0,1\}^q} \tilde{B}_{t_z} |\varphi_z\rangle \end{aligned}$$



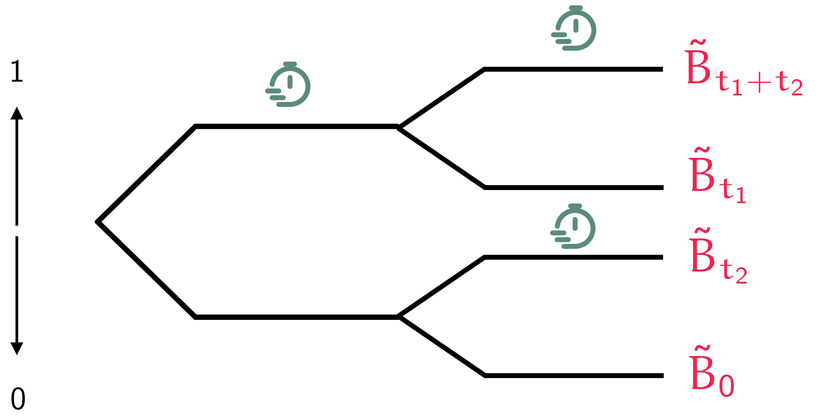
Diffusive Growth of Distinguishability

$$
\begin{aligned}
|\Delta_1^{(q)}\rangle &\coloneqq |\psi^{(q)}(h_1,\dots,h_m)\rangle - |\psi^{(q)}(0,\dots,h_m)\rangle \\
&= \sum_{z\in\{0,1\}^q} (e^{-ih_1\sum_{j:z_j=1} t_j} - 1)|\varphi_z\rangle \\
&=: \sum_{z\in\{0,1\}^q} \tilde{B}_{t_z}|\varphi_z\rangle
\end{aligned}
$$



Diffusive Growth of Distinguishability

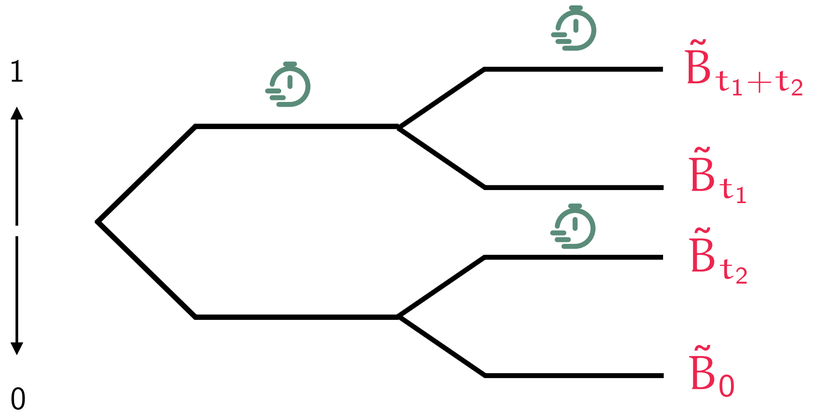
$$
\begin{aligned}
|\Delta_1^{(q)}\rangle &\coloneqq |\psi^{(q)}(h_1,\dots,h_m)\rangle - |\psi^{(q)}(0,\dots,h_m)\rangle \\
&= \sum_{z\in\{0,1\}^q} (e^{-ih_1\sum_{j:z_j=1} t_j} - 1)|\varphi_z\rangle \\
&=: \sum_{z\in\{0,1\}^q} \tilde{B}_{t_z}|\varphi_z\rangle
\end{aligned}
$$



- Suppose we have $\tilde{B}_t$ is a Brownian process. Then $\|\Delta_1^{(q)}\|_2^2$ is the variance of some linear observable.

Diffusive Growth of Distinguishability

$$
\begin{aligned}
|\Delta_1^{(q)}\rangle &\coloneqq |\psi^{(q)}(h_1,\dots,h_m)\rangle - |\psi^{(q)}(0,\dots,h_m)\rangle \\
&= \sum_{z\in\{0,1\}^q} (e^{-ih_1\sum_{j:z_j=1} t_j} - 1)|\varphi_z\rangle \\
&=: \sum_{z\in\{0,1\}^q} \tilde{B}_{t_z}|\varphi_z\rangle
\end{aligned}
$$



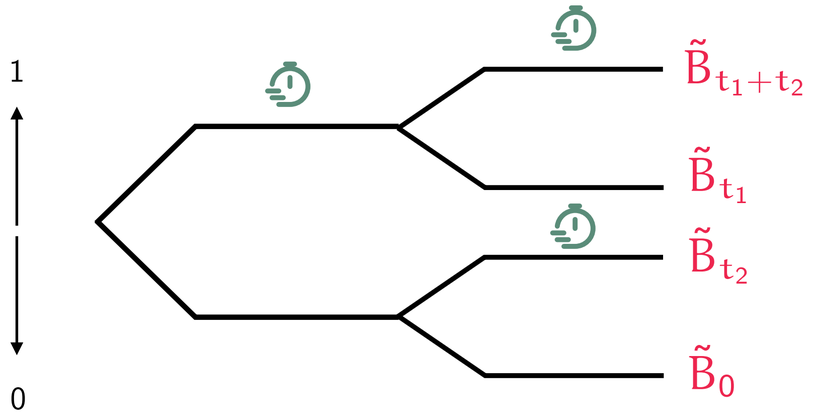
- Suppose we have $\tilde{B}_t$ is a Brownian process. Then $\|\Delta_1^{(q)}\|_2^2$ is the variance of some linear observable.
- This grows diffusively!

$$
\||\Delta_1^{(q)}\rangle\|_2^2 \overset{?}{=} \sum_{j=1}^{q} t_j \langle\psi^{(j-1)}|\Pi_1|\psi^{(j-1)}\rangle
$$

Accumulated time in subspace 1

Diffusive Growth of Distinguishability

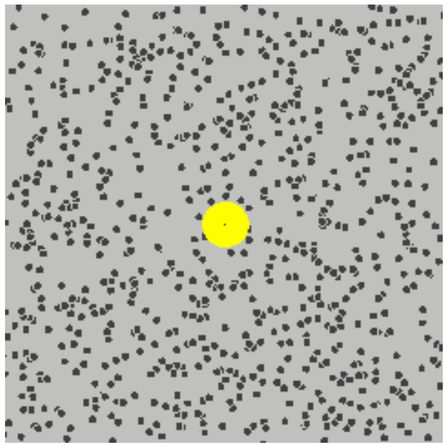
$$|\Delta_1^{(q)}\rangle \coloneqq |\psi^{(q)}(h_1,\dots,h_m)\rangle - |\psi^{(q)}(0,\dots,h_m)\rangle$$
$$= \sum_{z\in\{0,1\}^q} (e^{-ih_1\sum_{j:z_j=1} t_j} - 1)|\varphi_z\rangle$$
$$=: \sum_{z\in\{0,1\}^q} \tilde{B}_{t_z}|\varphi_z\rangle$$



- Suppose we have $\tilde{B}_t$ is a Brownian process. Then $\|\Delta_1^{(q)}\|_2^2$ is the variance of some linear observable.
- This grows diffusively!

$$\||\Delta_1^{(q)}\rangle\|_2^2 \overset{?}{=} \sum_{j=1}^{q} t_j \langle\psi^{(j-1)}|\Pi_1|\psi^{(j-1)}\rangle$$

Accumulated time in subspace 1



Brownian motion: mean-squared distance grows linearly with time

Connection to Brownian Process

- Recall our actual random process is defined through: $\tilde{B}_t = e^{-ih_1 t} - 1$

Connection to Brownian Process

- Recall our actual random process is defined through: $\tilde{B}_t = e^{-ih_1 t} - 1$
- Define the covariance kernel $K(s,t) \coloneqq \mathbb{E}\, \tilde{B}_s \tilde{B}_t$

Connection to Brownian Process

- Recall our actual random process is defined through: $\tilde{B}_t = e^{-ih_1 t} - 1$
- Define the covariance kernel $K(s,t) \coloneqq \mathbb{E}\,\tilde{B}_s \tilde{B}_t$

Lemma (Kernel bound).

For every $t_1, \ldots, t_q \geqslant 0$ and $c_1, \ldots, c_n \in \mathbb{C}$ it holds that

$$\sum_{i,j=1}^{q} c_i^* c_j K(t_i, t_j) \leqslant \pi\varepsilon^2 \sum_{i,j=1}^{q} c_i^* c_j \min(t_i, t_j)$$

Connection to Brownian Process

- Recall our actual random process is defined through: $\tilde{B}_t = e^{-ih_1 t} - 1$
- Define the covariance kernel $K(s,t) \coloneqq \mathbb{E}\,\tilde{B}_s \tilde{B}_t$

Lemma (Kernel bound).

For every $t_1, \ldots, t_q \geqslant 0$ and $c_1, \ldots, c_n \in \mathbb{C}$ it holds that

$$\sum_{i,j=1}^{q} c_i^* c_j K(t_i, t_j) \leqslant \pi\varepsilon^2 \sum_{i,j=1}^{q} c_i^* c_j \min(t_i, t_j)$$

covariance of Brownian process!

Connection to Brownian Process

- Recall our actual random process is defined through: $\tilde{B}_t = e^{-ih_1 t} - 1$
- Define the covariance kernel $K(s,t) \coloneqq \mathbb{E}\, \tilde{B}_s \tilde{B}_t$

Lemma (Kernel bound).

For every $t_1, \ldots, t_q \geqslant 0$ and $c_1, \ldots, c_n \in \mathbb{C}$ it holds that

$$\sum_{i,j=1}^{q} c_i^* c_j K(t_i, t_j) \leqslant \pi\varepsilon^2 \sum_{i,j=1}^{q} c_i^* c_j \min(t_i, t_j)$$

covariance of Brownian process!

- This is the crucial (and only) place where we assume forward time access

Proof Checklist

1. ~~Construct the hardest possible random Hamiltonian H to distinguish from zero~~
2. ~~Check that it is still ϵ far, so the certification algorithm is successful~~
3. ~~Bound distinguishability as a function of total evolution time~~

New techniques needed: (i) hybrid method and (ii) compressed oracle method fail

Recap: (i) Efron-Stein to isolate Hamiltonian terms, (ii) kernel bound to relate amplitudes to Brownian process, (iii) diffusive growth of distinguishability

Overview

1. Background & Problem Statement
2. Main Results
3. Proof Sketch
4. Conclusion & Future Directions

Without time reversal...

- Hamiltonian property testing is restricted to SQL
- Amplitude estimation has classical scaling
- No robust quantum advantage in search

Future directions

- Hardness of reversing time in quantum experiments (e.g., local Hamiltonians)
- Robustness of quantum oracle speedups to imperfections
- Applications of our method to other quantum lower bounds