The Power of Time Reversal in Hamiltonian Property Testing

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Tom Gur (Cambridge) **Angus Lowe** (MIT)

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arXiv:2610.04996

Overview

1. Background & Problem Statement
2. Main Results
3. Proof Sketch
4. Conclusion & Future Directions

Time reversal in quantum information

Time reversal is a key step in many tasks across QI

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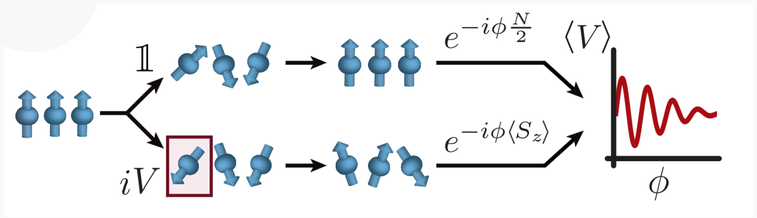
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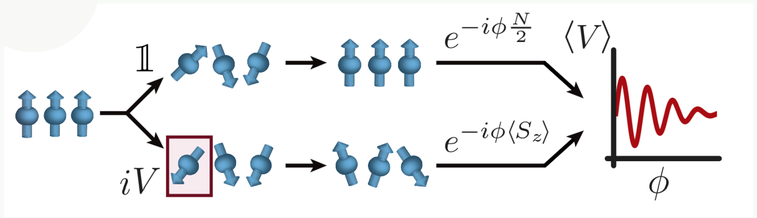


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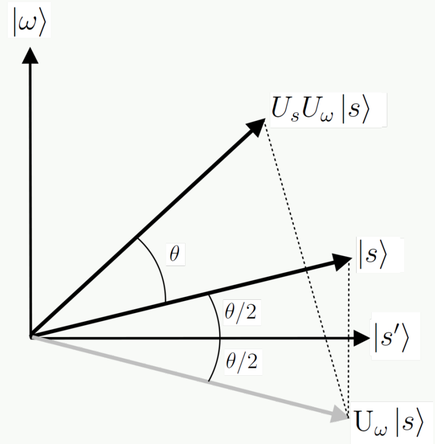
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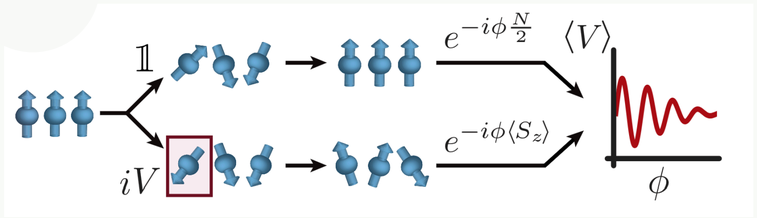


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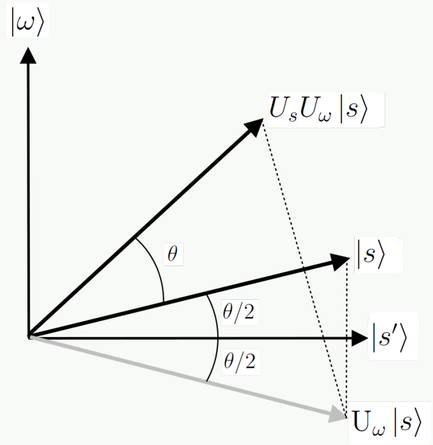
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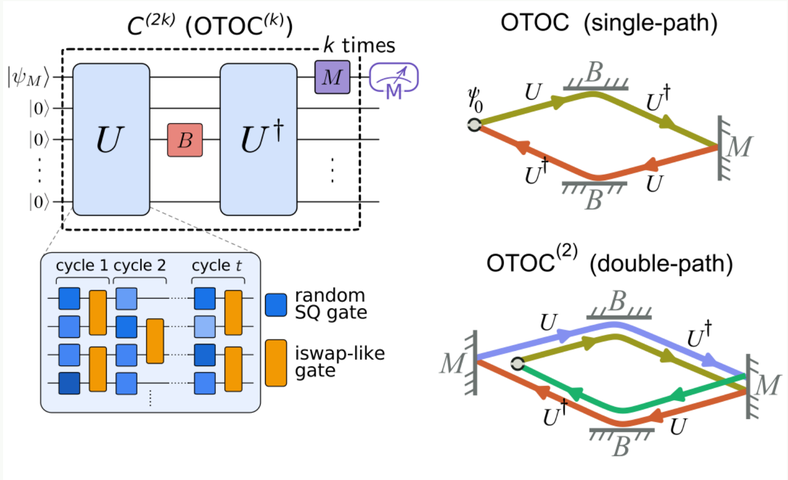
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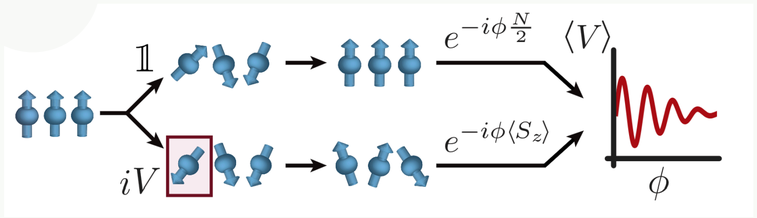
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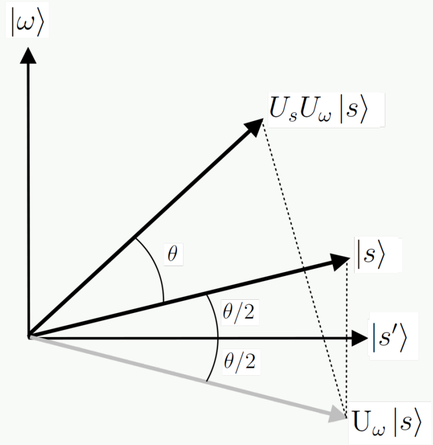
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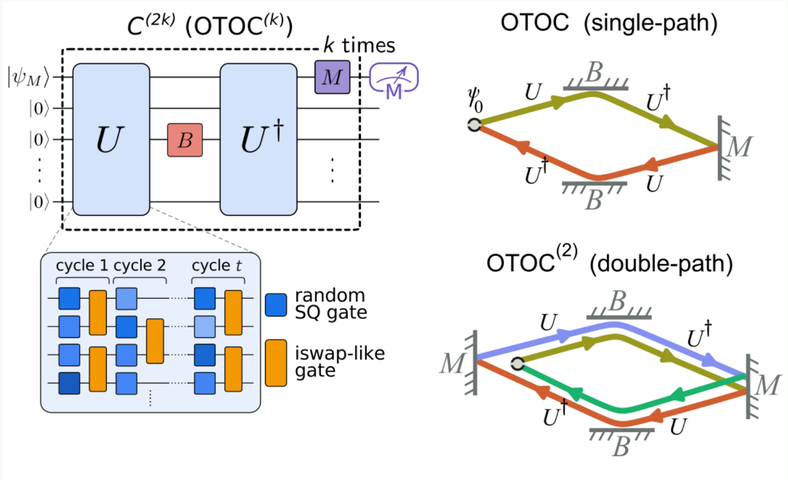
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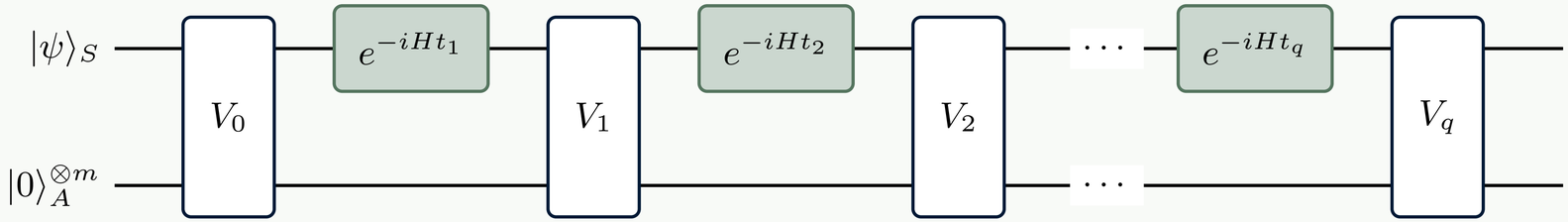


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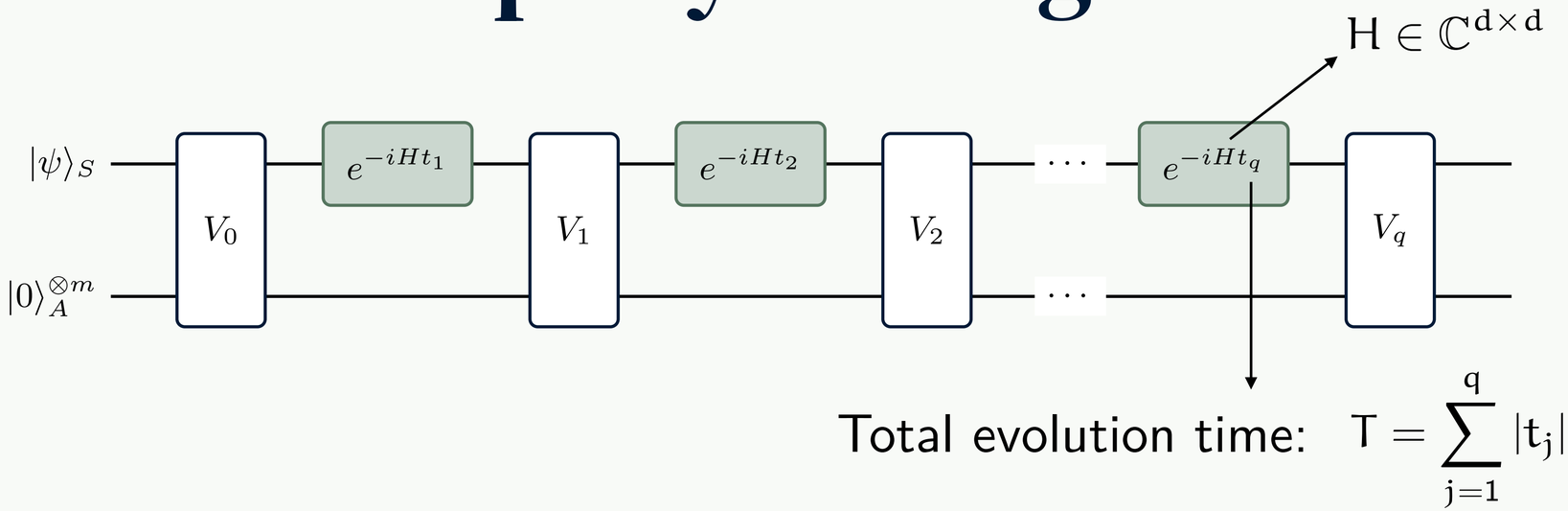
Q: *When is time reversal truly required*?

[1] Kobrin et al. A Universal Protocol for Quantum-Enhanced Sensing via Information Scrambling.
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Hamiltonian Property Testing

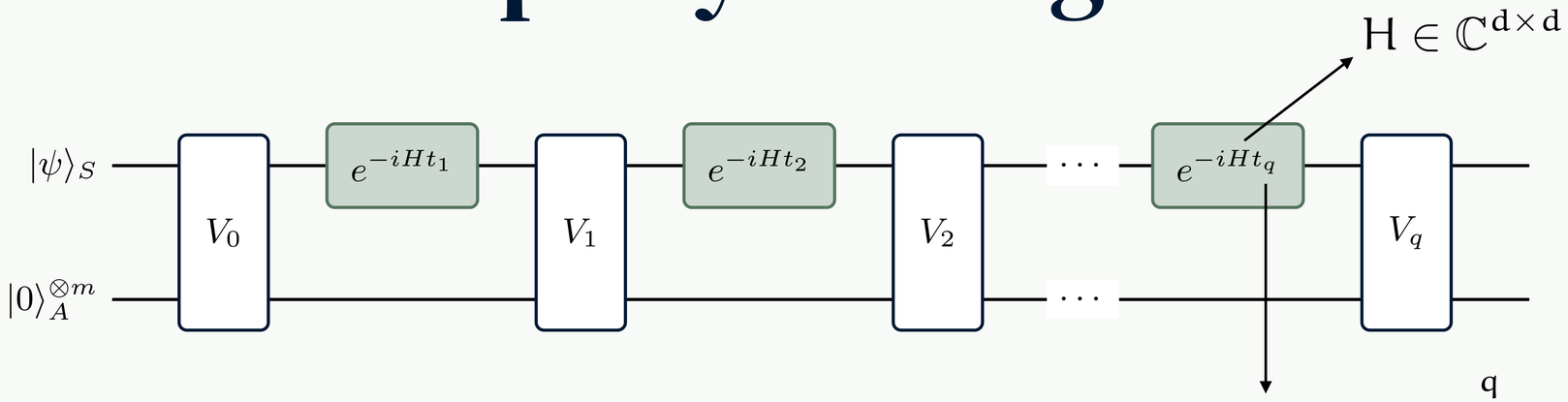


Hamiltonian Property Testing



Total evolution time: $T = \sum_{j=1}^{q} |t_j|$

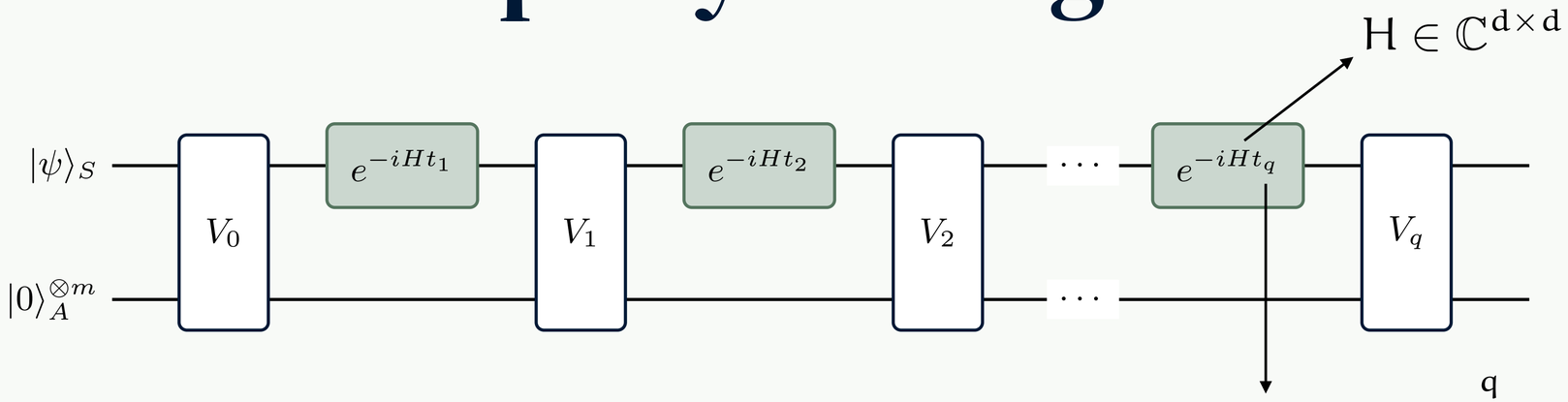
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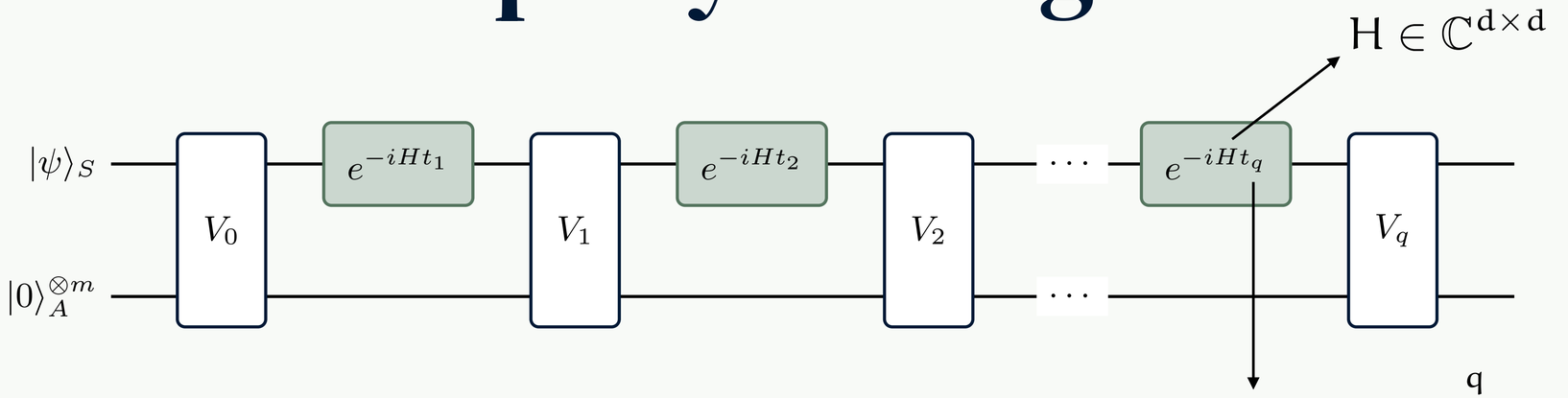
Let H_0 be a known target Hamiltonian. Decide:

(i) if $H = H_0$, accept w.h.p.

(ii) if $\frac{1}{\sqrt{d}}\|H - H_0\|_F > \varepsilon$, reject w.h.p.

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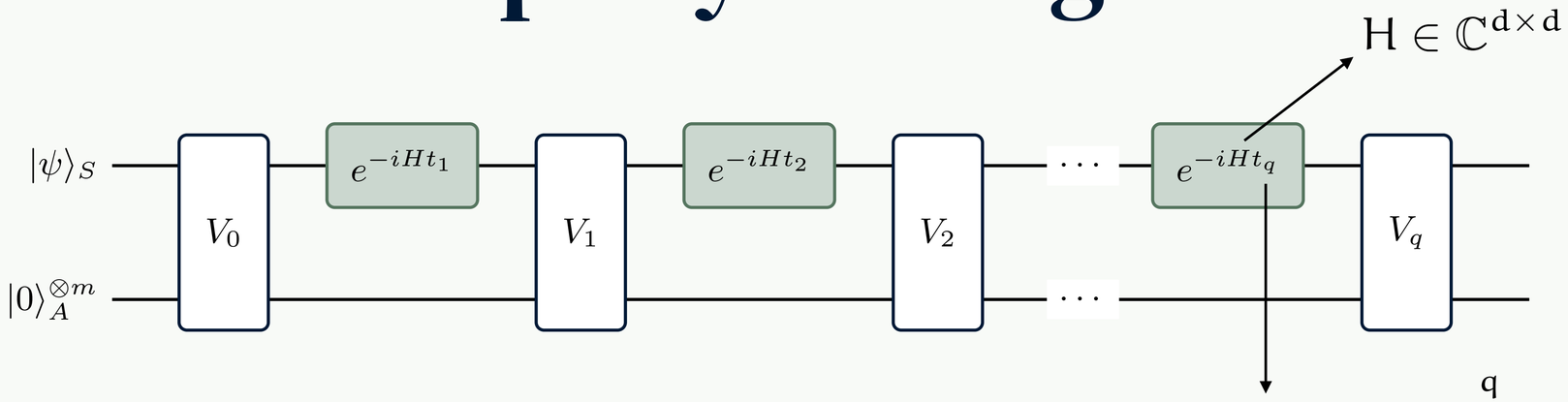
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Average-case rate of distinguishability

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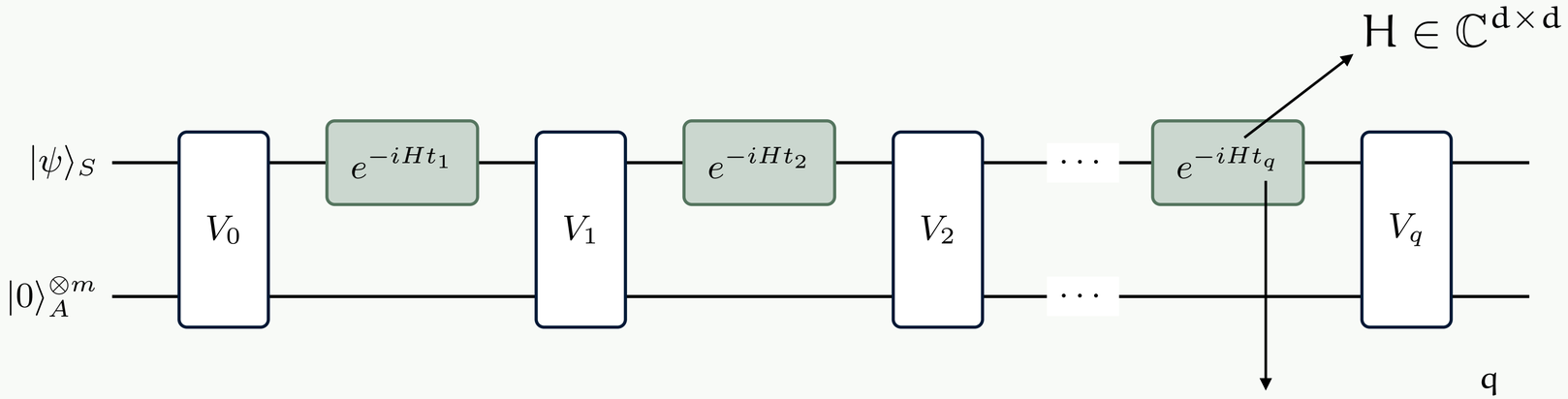
Let k be a positive integer. Decide:

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(ii) if H is ε-far from k-local, reject w.h.p.

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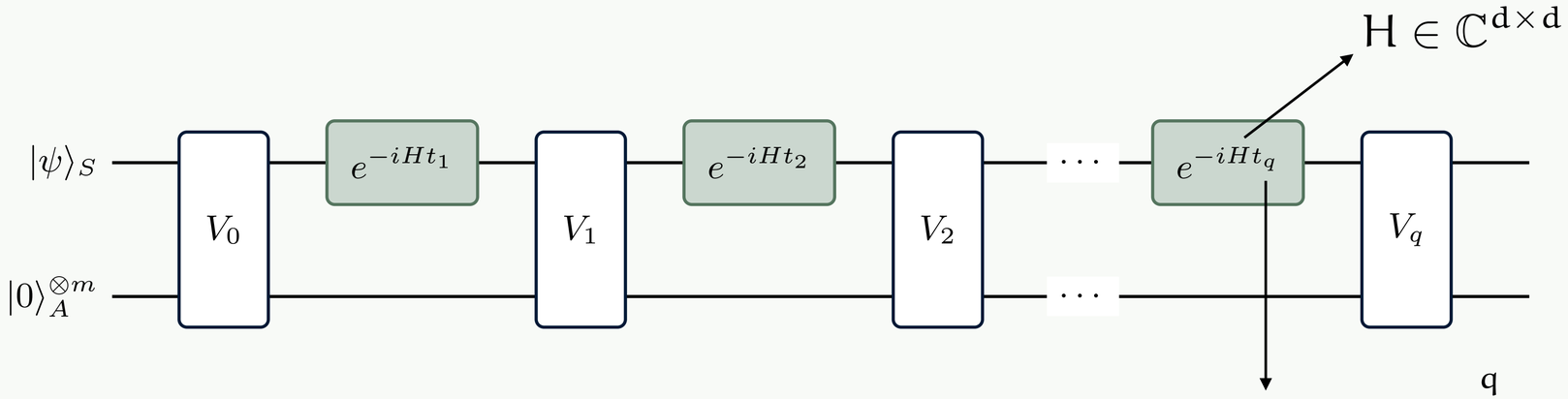
Prior Work



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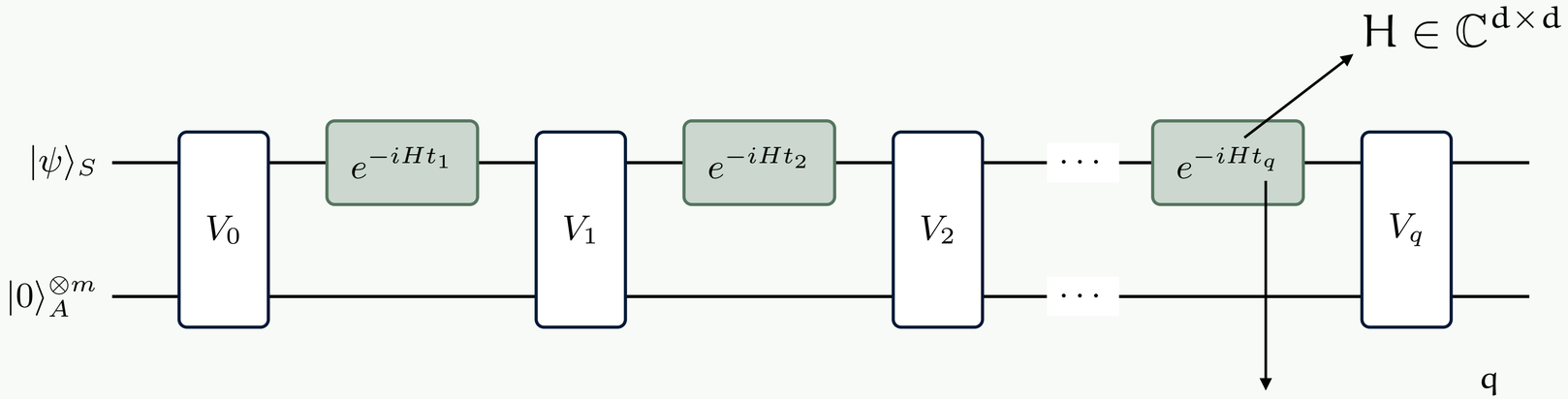


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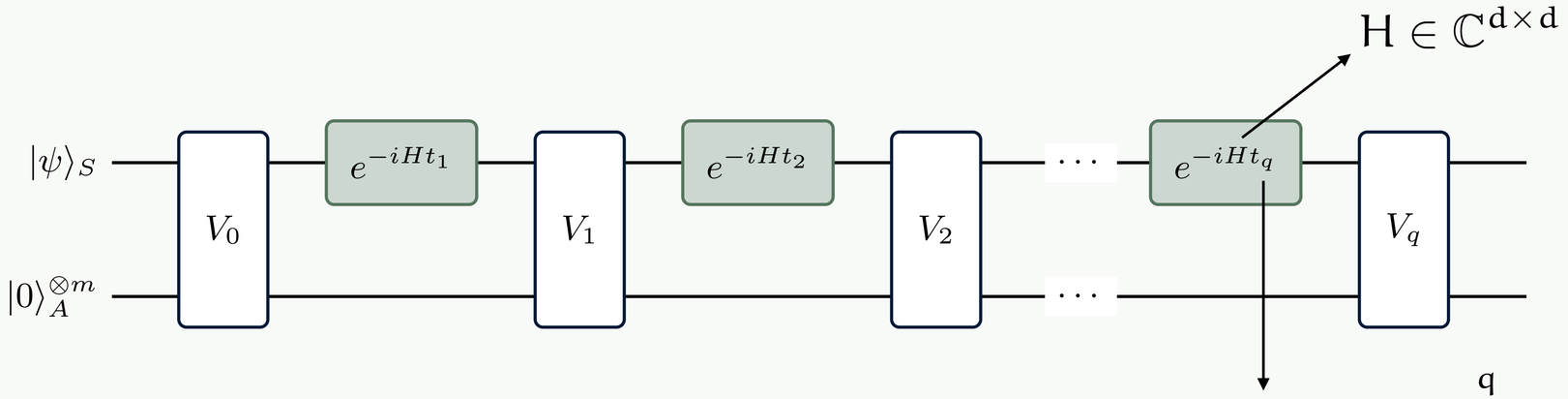


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Q: is time reversal required for Heisenberg-limited scaling?

[KL'25] Kallaugher, Liang. Hamiltonian locality testing via Trotterized postselection.
[Gao+'26] Gao et al. Quantum Hamiltonian certification.
[ST'25] Sinha, Tong. Improved Hamiltonian Learning and Sparsity Testing through Bell Sampling.

Prior Work

Problem (Amplitude Estimation).

Given U on $n+1$ qubits such that

$$U|0^{n+1}\rangle = a|\phi_1\rangle|1\rangle + \sqrt{1-a^2}|\phi_0\rangle|0\rangle$$

output $\hat{a}$ such that $|\hat{a}-a| < \varepsilon$ with high probability, using as few queries to U and $U^\dagger$ as possible.

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A: No, discrete queries are different. ([HTFS'23] vs [DOS'24].)

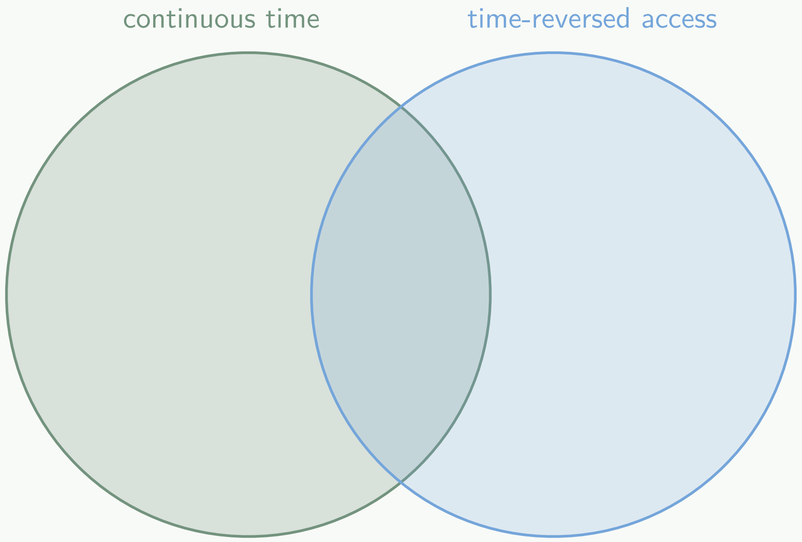
[BBHT'98] Brassard et al. Quantum Amplitude amplification and estimation.
[TW'26] Tang, Wright. Amplitude amplification and estimation require inverses.
[HTFS'23] Huang et al. Learning many-body Hamiltonians with Heisenberg-limited scaling.
[DOS'24] Dutkiewicz, O'Brien, Schuster. The advantage of quantum control in many-body Hamiltonian learning.

Key Technical Challenge

- We need a quantum lower bound technique that:
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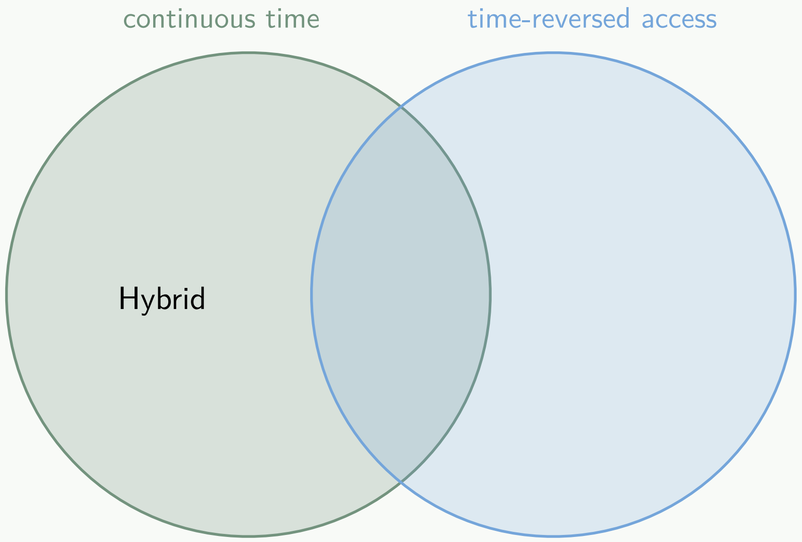
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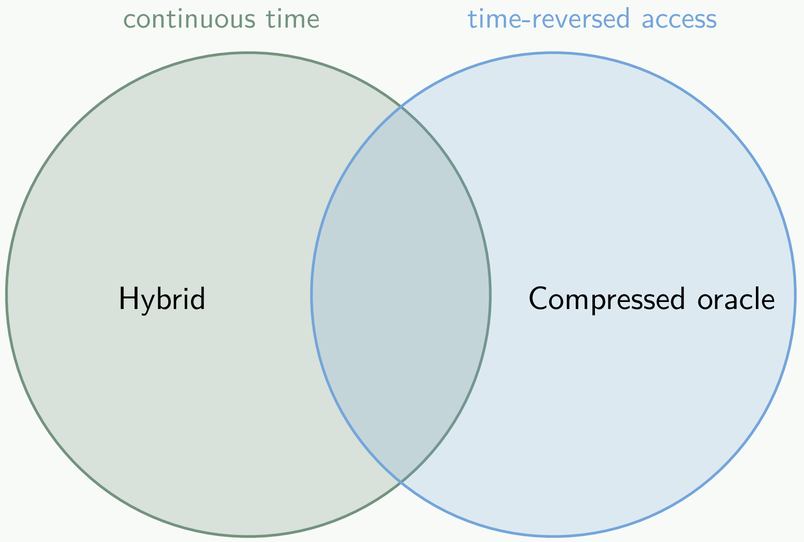
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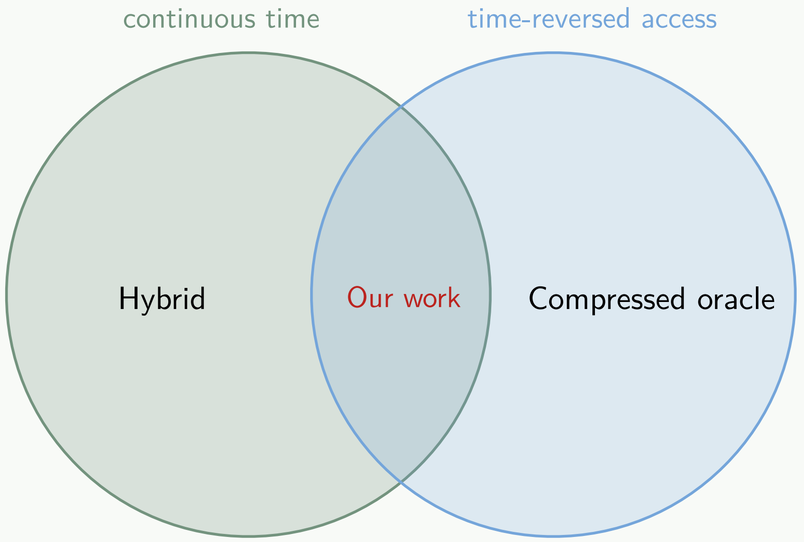
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Theorem (Hamiltonian certification without time reversal [This work]).

Any quantum algorithm without time reversal for Hamiltonian certification requires a total evolution time

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- And for certification with arbitrary Schatten p-norm (from average- to worst-case distinguishability):

$$T = \Theta\left(\min\left\{\frac{1}{\varepsilon^p}, \frac{d^{1-1/p}}{\varepsilon}\right\}\right)$$

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- Same lower bound holds for locality testing.

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Amplitude Estimation

Corollary (Amplitude estimation without inverses [This work]).

Given only the ability to apply U and not its inverse, any algorithm for amplitude estimation requires

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- Extends results in [TW'26] to fractional queries
- Improved lower bound in the high-precision regime where $\varepsilon \ll d^{-1/2}$

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[TW'26] Tang, Wright. Amplitude amplification and estimation require inverses.

Quantum Search with Coherent Errors

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$$\mathrm{O}_{\mathrm{f}}^{\theta} = \sum_{x=1}^{d} e^{i\theta_x f(x)} |x\rangle\langle x|$$

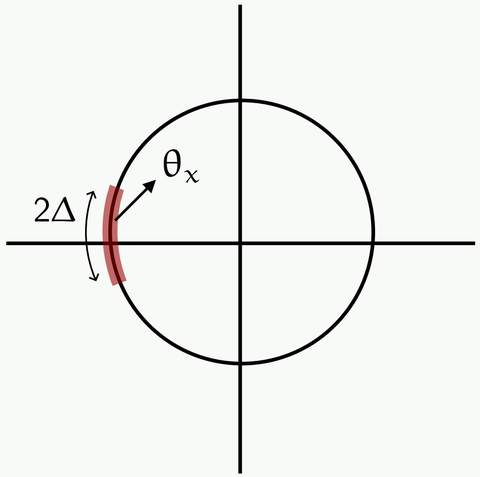
for some unknown angles $\theta_x \in [\pi - \Delta, \pi + \Delta]$ and unknown f that marks $\Theta(1)$ elements.

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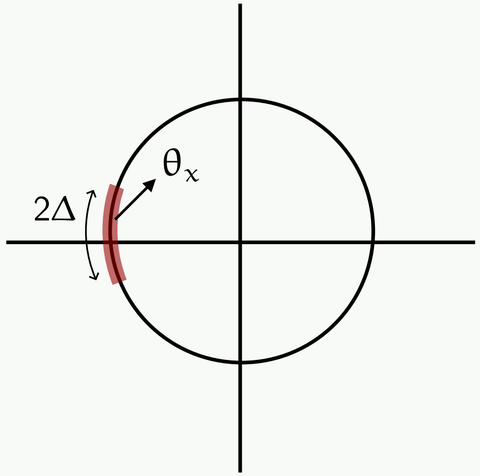


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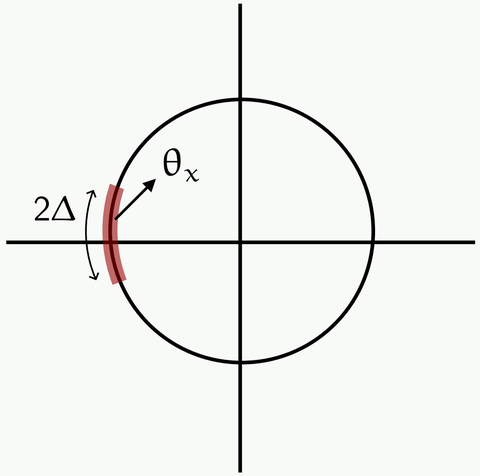
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Theorem (No-go for quantum search [This work]).

Given only the ability to apply O_f^θ and not its inverse, any algorithm for search requires at least $\Omega(d\Delta)$ applications of O_f^θ.

- By contrast, with inverse access, $O(\sqrt{d})$ applications suffice [HMdW'03]

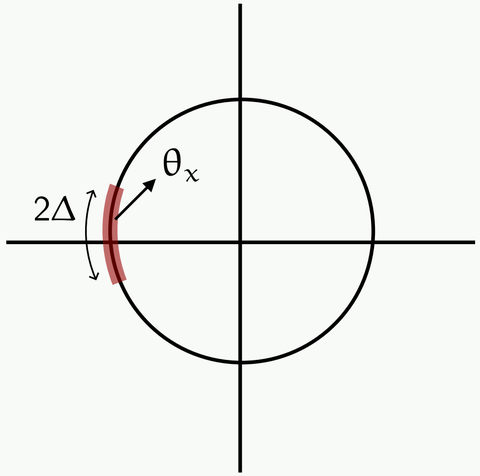
[HMdW'03] Hoyer, Mosca, de Wolfe. Quantum search with bounded-error inputs.
[RS'12] Regev, Schiff. Impossibility of a Quantum Speed-up with a Faulty Oracle.

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for some unknown angles $\theta_x \in [\pi - \Delta, \pi + \Delta]$ and unknown f that marks $\Theta(1)$ elements.



Theorem (No-go for quantum search [This work]).

Given only the ability to apply O_f^θ and not its inverse, any algorithm for search requires at least $\Omega(d\Delta)$ applications of O_f^θ.

- By contrast, with inverse access, $O(\sqrt{d})$ applications suffice [HMdW'03]
- Distinct from no-go results based on noisy channels [RS'12]

[HMdW'03] Hoyer, Mosca, de Wolfe. Quantum search with bounded-error inputs.
[RS'12] Regev, Schiff. Impossibility of a Quantum Speed-up with a Faulty Oracle.

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3. Proof Sketch
4. Conclusion & Future Directions

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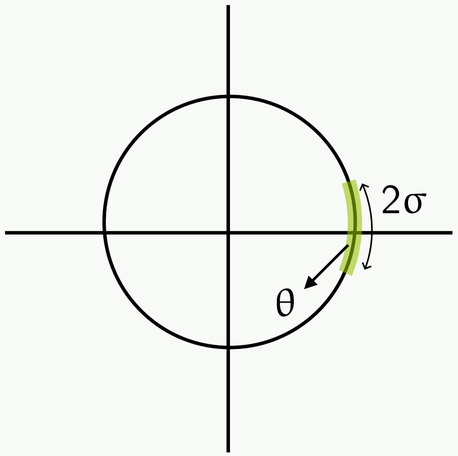
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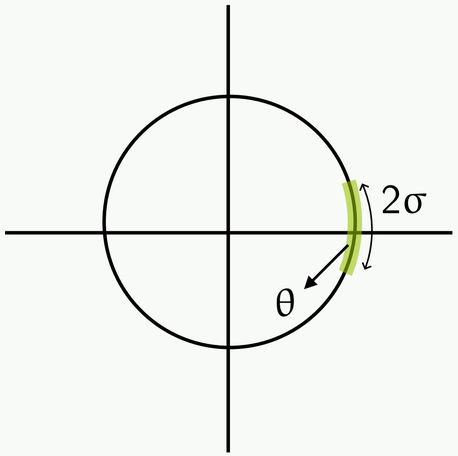


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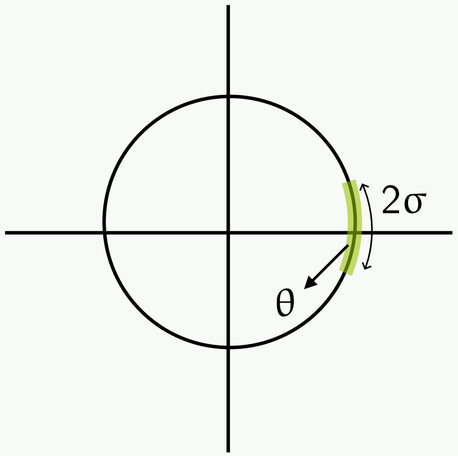
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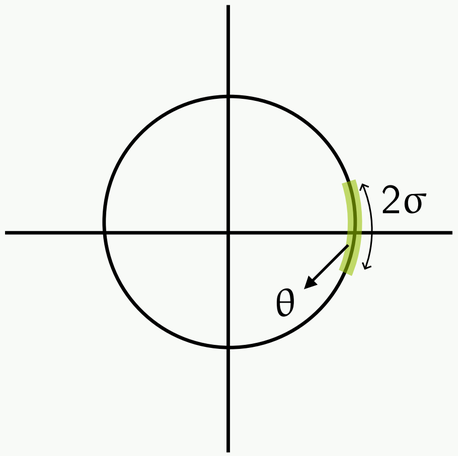
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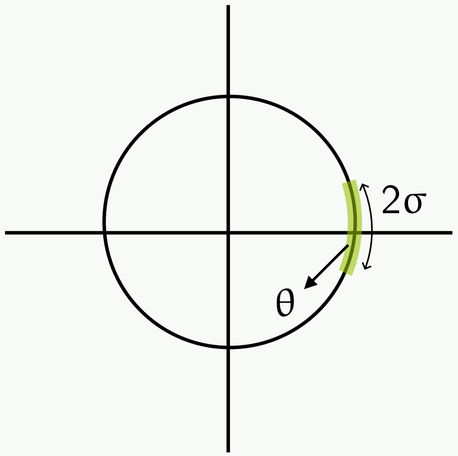
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Theorem.

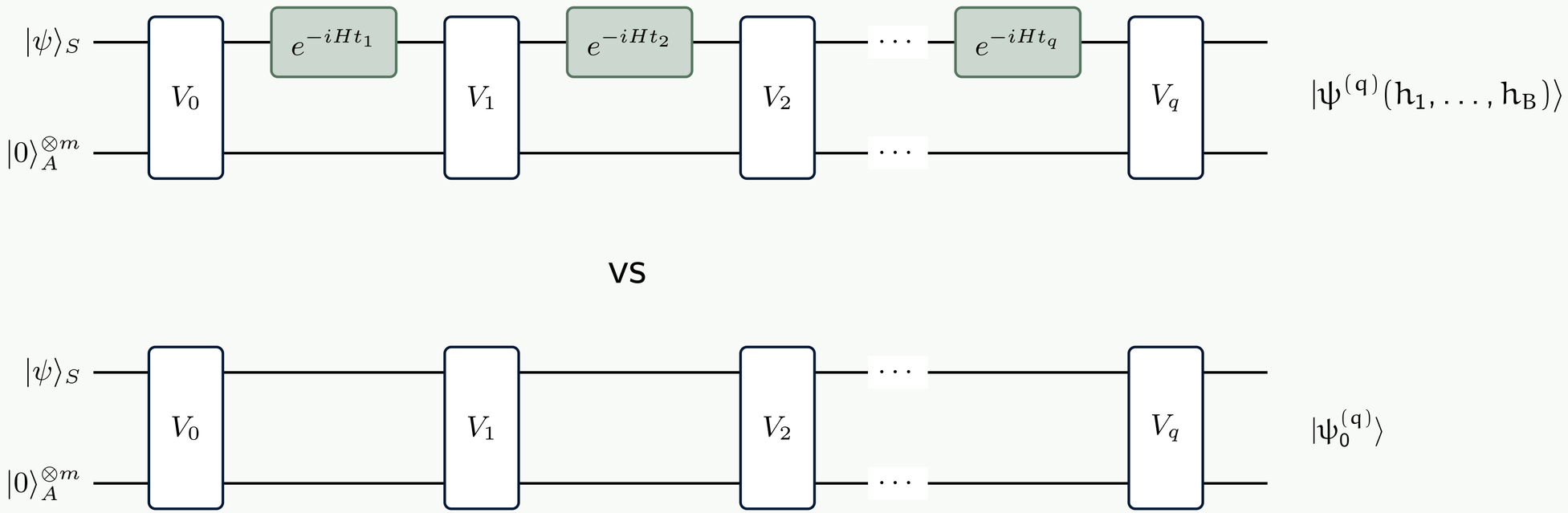
Any quantum algorithm to distinguish H from zero without time reversal requires a total evolution time $T = \Omega(1/\varepsilon^2)$.

Proof Checklist

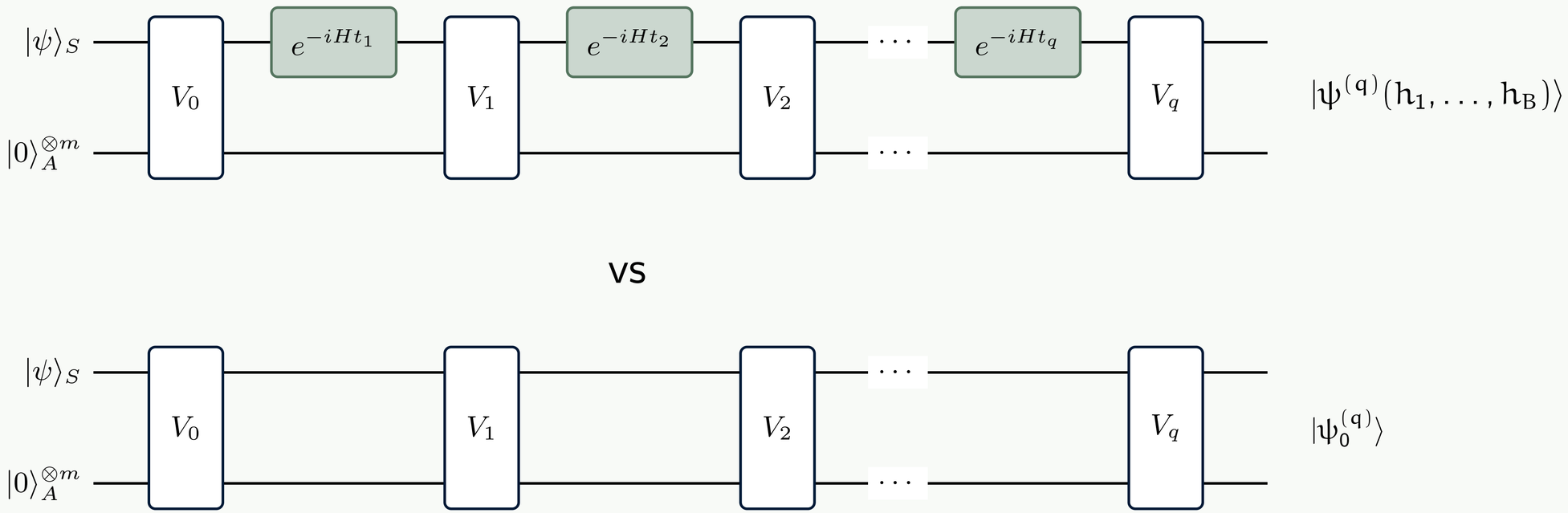
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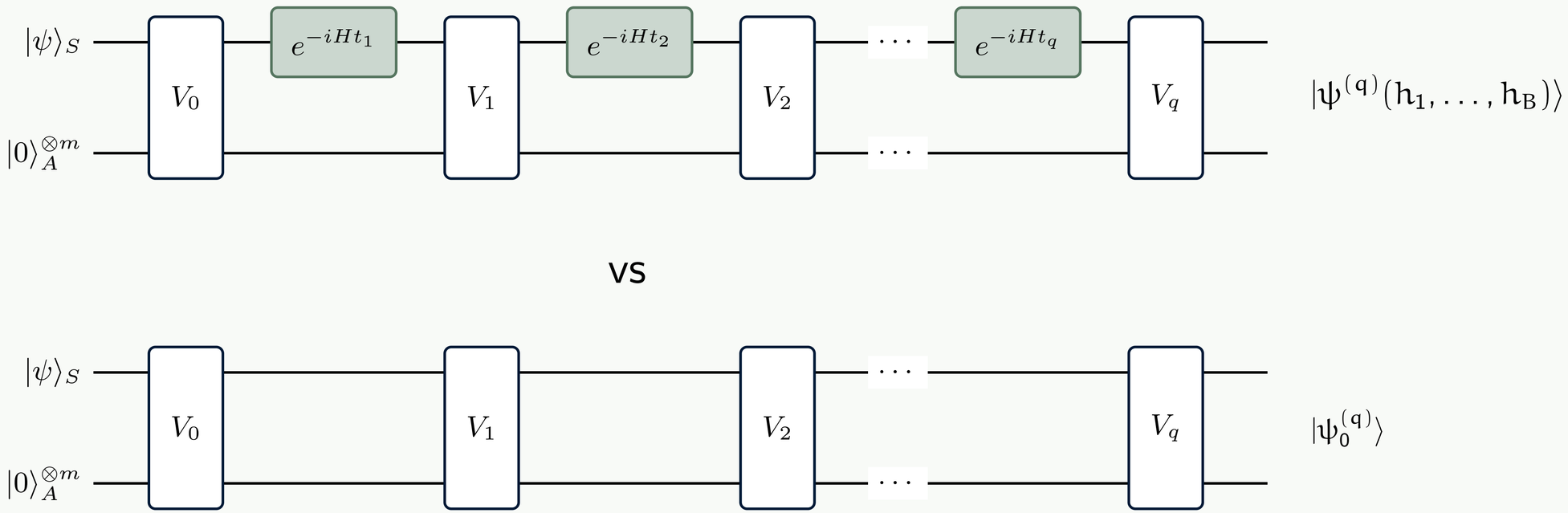


Bounding distinguishability



Look at the transition probability $\mathfrak{p} \coloneqq \mathbb{E}\,\|(I - |\psi_0^{(q)}\rangle\langle\psi_0^{(q)}|)|\psi^{(q)}(h_1,\ldots,h_B)\rangle\|_2^2$

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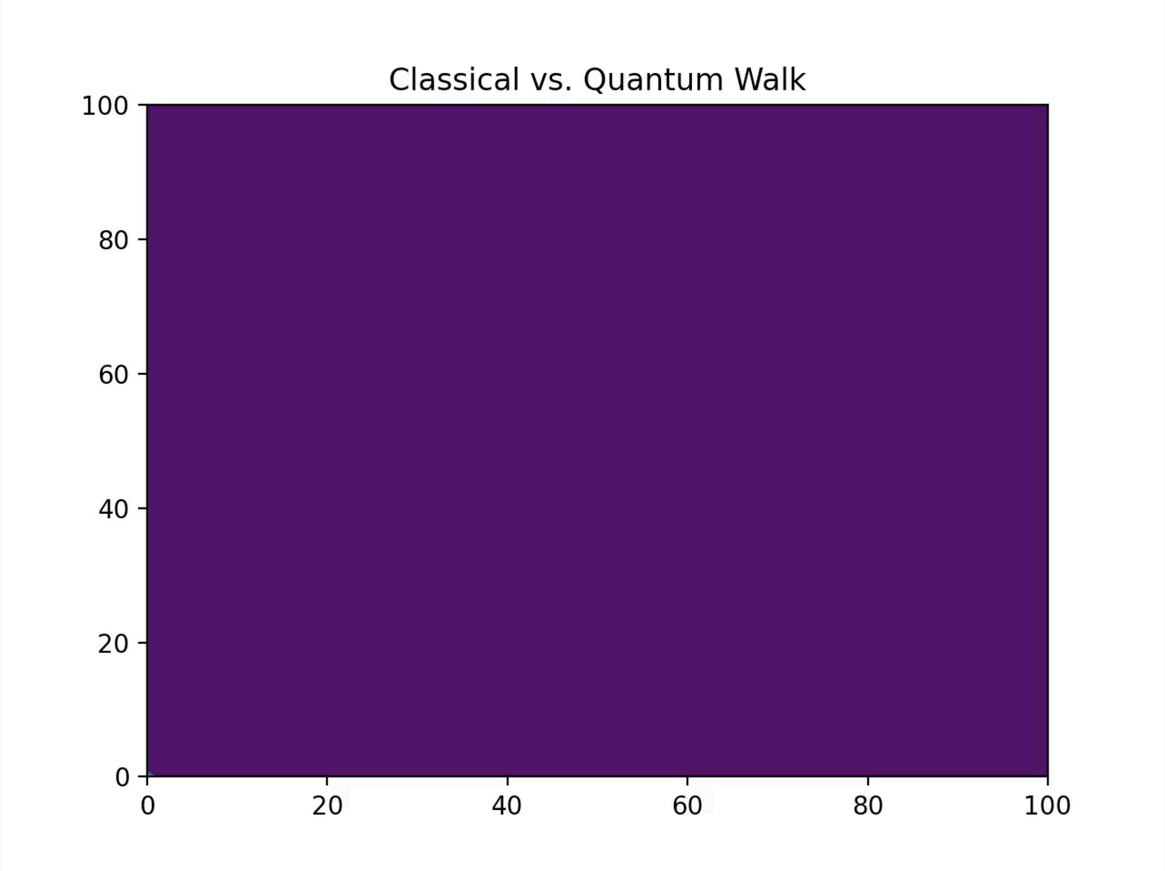


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Q: Can we show $\mathfrak{p} \leqslant \mathrm{O}(\varepsilon^2 \mathrm{T})$?

Note: *not* $\mathrm{O}(\varepsilon^2 \mathrm{T}^2)$

Diffusive vs ballistic growth of *distinguishability*


Classical vs. Quantum Walk
100
80
60
40
20
0
0
20
40
60
80
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First Attempt: Hybrid Method

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- Indeed, leads to ballistic growth upper bound, $\mathfrak{p} \leqslant O(\varepsilon^2 T^2)$

Isolate Hamiltonian Terms

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- Similar reasoning leads to:

$$\mathfrak{p} \leqslant \sum_{i=1}^{B} \||\Delta_i^{(q)}\rangle\|_2^2, \quad |\Delta_i^{(q)}\rangle \coloneqq |\psi^{(q)}(h_1,\ldots,h_i,\ldots,h_m)\rangle - |\psi^{(q)}(h_1,\ldots,0,\ldots,h_m)\rangle$$

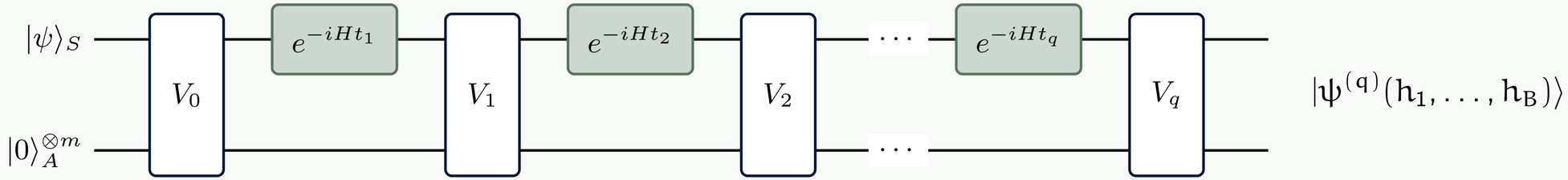
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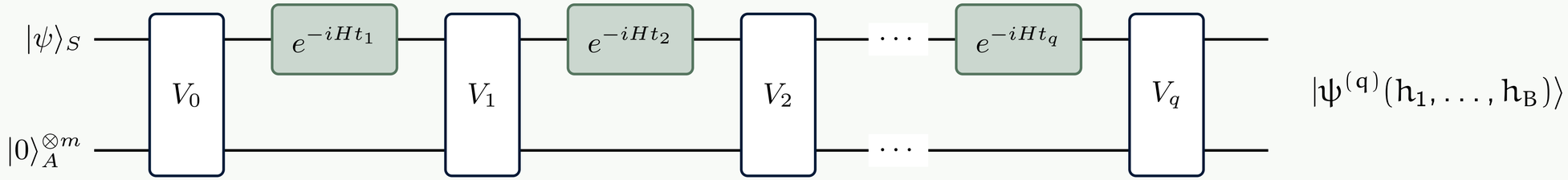
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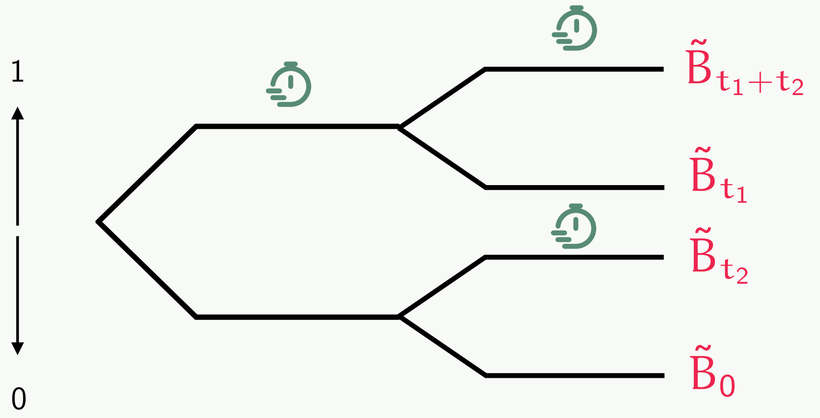
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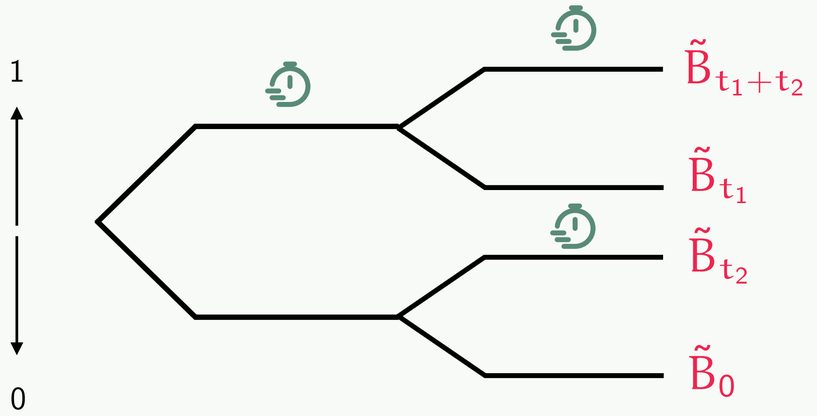


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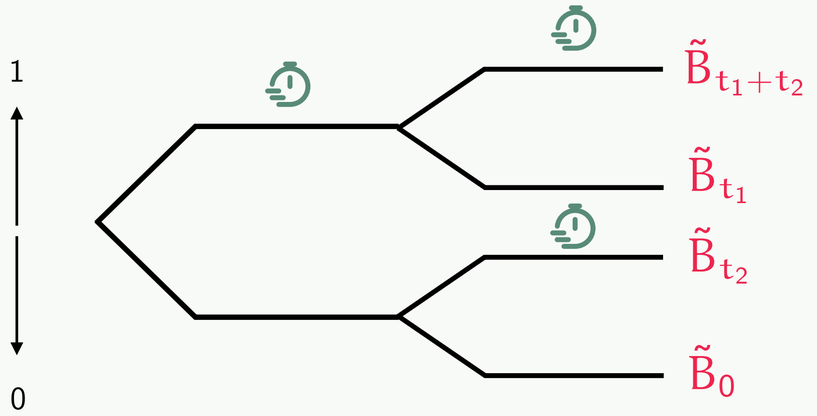
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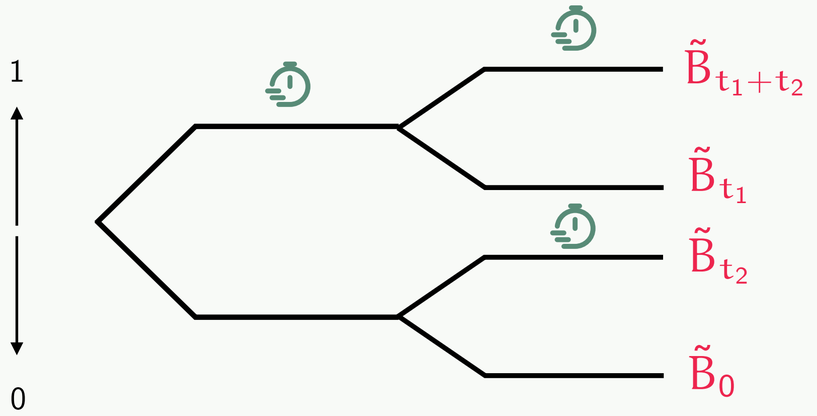
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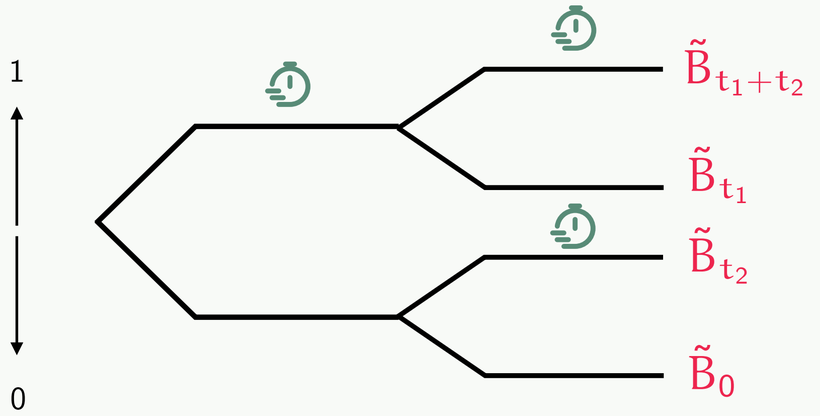
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Accumulated time in subspace 1

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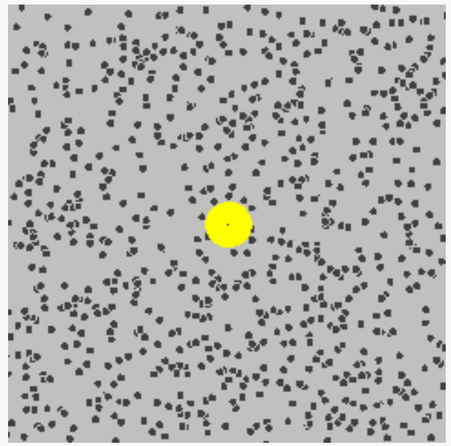
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Accumulated time in subspace 1



Brownian motion: mean-squared distance grows linearly with time

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For every $t_1, \ldots, t_q \geqslant 0$ and $c_1, \ldots, c_n \in \mathbb{C}$ it holds that

$$\sum_{i,j=1}^{q} c_i^* c_j K(t_i, t_j) \leqslant \pi\varepsilon^2 \sum_{i,j=1}^{q} c_i^* c_j \min(t_i, t_j)$$

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- This is the crucial (and only) place where we assume forward time access
- Proof: whiteboard

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Recap: (i) Efron-Stein to isolate Hamiltonian terms, (ii) kernel bound to relate amplitudes to Brownian process, (iii) diffusive growth of distinguishability

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Without time reversal...

- Hamiltonian property testing is restricted to SQL
- Amplitude estimation has classical scaling
- No robust quantum advantage in search

Future directions

- Hardness of reversing time in quantum experiments
- From generic to local Hamiltonians
- Robustness of quantum oracles to noise?
- Applications of our method to other quantum lower bounds