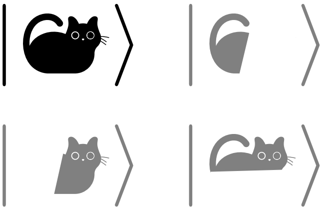
Randomized Truncation

(of Quantum States)

Aram Harrow (MIT) *Angus Lowe* (MIT) Freek Witteveen (CWI/QuSOFT)

March 10, 2026



arXiv:2510.08518

TL;DR

Main Result: *we solve for the optimal randomized truncation to a quantum state.*

Open Question: *applications?*

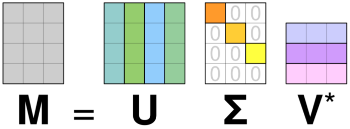
What is the best low-rank approximation to a matrix, M?

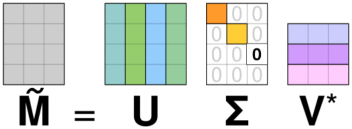
$$\begin{aligned} \text{minimize} \quad & \|M - \tilde{M}\|_\mathsf{F} \\ \text{subject to} \quad & \text{rank}(\tilde{M}) \leq k \end{aligned}$$

What is the best low-rank approximation to a matrix, M?

$$\begin{aligned} \text{minimize} \quad & \|M - \tilde{M}\|_\mathsf{F} \\ \text{subject to} \quad & \operatorname{rank}(\tilde{M}) \leq k \end{aligned}$$

$\longrightarrow$ take the top k singular values [EY'36]



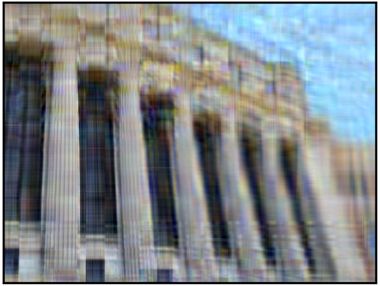


What is the best low-rank approximation to a matrix, M?

$$\begin{aligned} \text{minimize} \quad & \|M - \tilde{M}\|_{\mathsf{F}} \\ \text{subject to} \quad & \operatorname{rank}(\tilde{M}) \le k \end{aligned}$$

$\longrightarrow$ take the top k singular values [EY'36]

Original

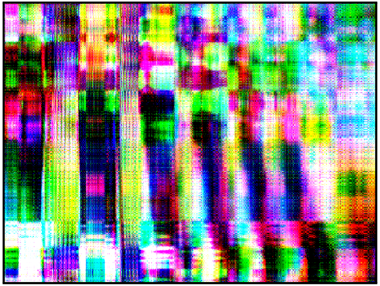

Top 10 SVs

What is the best low-rank approximation to a matrix, M?

$$\begin{aligned} \text{minimize} \quad & \|M - \tilde{M}\|_{\mathsf{F}} \\ \text{subject to} \quad & \operatorname{rank}(\tilde{M}) \le k \end{aligned}$$

$\longrightarrow$ take the top k singular values [EY'36]

Original

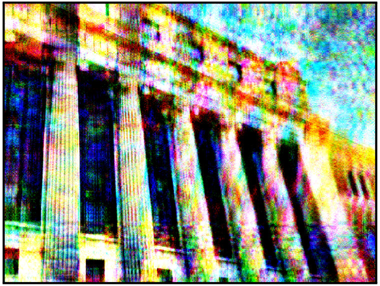

Random 10 SVs

What is the best low-rank approximation to a matrix, M?

$$\begin{aligned} \text{minimize} \quad & \|M - \tilde{M}\|_{\mathsf{F}} \\ \text{subject to} \quad & \operatorname{rank}(\tilde{M}) \leq k \end{aligned}$$

$\longrightarrow$ take the top k singular values [EY'36]

Original



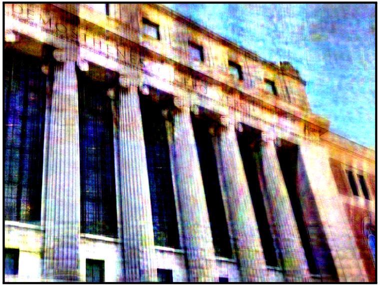
Random 10 SVs, 10 samples

What is the best low-rank approximation to a matrix, M?

$$
\begin{aligned}
\text{minimize} \quad & \|M - \tilde{M}\|_{\mathsf{F}} \\
\text{subject to} \quad & \operatorname{rank}(\tilde{M}) \leq k
\end{aligned}
$$

$\longrightarrow$ take the top k singular values [EY'36]

Original



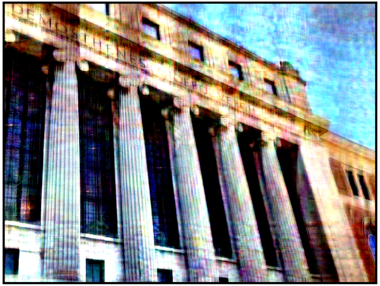
Random 10 SVs, 100 samples

What is the best low-rank approximation to a matrix, M?

$$\begin{aligned} \text{minimize} \quad & \|M - \tilde{M}\|_{\mathrm{F}} \\ \text{subject to} \quad & \operatorname{rank}(\tilde{M}) \le k \end{aligned}$$

$\longrightarrow$ take the top k singular values [EY'36]

Original



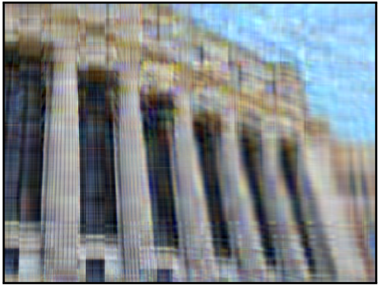
Random 10 SVs, 200 samples

What is the best low-rank approximation to a matrix, M?

$$\begin{aligned} \text{minimize} \quad & \|M - \tilde{M}\|_{\mathsf{F}} \\ \text{subject to} \quad & \operatorname{rank}(\tilde{M}) \le k \end{aligned}$$

$\longrightarrow$ take the top k singular values [EY'36]

Original



Top 10 SVs

What is the best low-Schmidt-rank approximation to an entangled state, $|\psi\rangle$?

$$\begin{aligned} \text{maximize} \quad & F(|\psi\rangle, |\tilde{\psi}\rangle) \\ \text{subject to} \quad & |\tilde{\psi}\rangle \in \mathsf{SR}_k \end{aligned}$$

What is the best low-Schmidt-rank approximation to an entangled state, $|\psi\rangle$?

$$\begin{aligned} \text{maximize} \quad & F(|\psi\rangle, |\tilde{\psi}\rangle) \\ \text{subject to} \quad & |\tilde{\psi}\rangle \in \mathrm{SR}_k \end{aligned}$$

Same problem in disguise!

$$|\psi\rangle = \sum_{i=1}^{r} \lambda_i |u_i\rangle \otimes |v_i\rangle \quad \longrightarrow \quad |\tilde{\psi}\rangle \propto \sum_{i=1}^{k} \lambda_i |u_i\rangle \otimes |v_i\rangle$$

$$F^\star = \|\lambda\|_{(k)} := \sqrt{\sum_{i=1}^{k} \lambda_i^2}$$

What is the best low-Schmidt-rank approximation to an entangled state, $|\psi\rangle$?

$$\begin{aligned} \text{maximize} \quad & F(|\psi\rangle, |\tilde{\psi}\rangle) \\ \text{subject to} \quad & |\tilde{\psi}\rangle \in \mathsf{SR}_k \end{aligned}$$

Same problem in disguise!

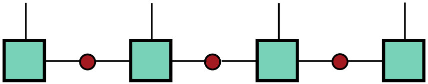
Application to matrix product states (MPS):

What is the best *low-Schmidt-rank* *approximation to an* *entangled* *state,* $|\psi\rangle$*?*

$$\begin{aligned} \text{maximize} \quad & F(|\psi\rangle, |\tilde{\psi}\rangle) \\ \text{subject to} \quad & |\tilde{\psi}\rangle \in \mathrm{SR}_k \end{aligned}$$

Same problem in disguise!

Application to matrix product states (MPS):

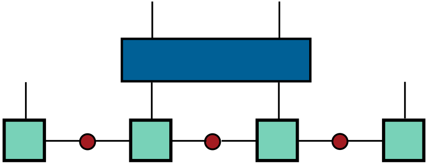


What is the best low-Schmidt-rank approximation to an entangled state, $|\psi\rangle$?

$$\begin{aligned} \text{maximize} \quad & F(|\psi\rangle, |\tilde{\psi}\rangle) \\ \text{subject to} \quad & |\tilde{\psi}\rangle \in \mathrm{SR}_k \end{aligned}$$

Same problem in disguise!

Application to matrix product states (MPS):

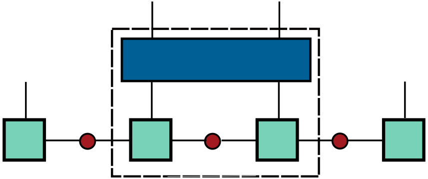


What is the best low-Schmidt-rank approximation to an entangled state, $|\psi\rangle$?

$$\begin{aligned} \text{maximize} \quad & F(|\psi\rangle, |\tilde{\psi}\rangle) \\ \text{subject to} \quad & |\tilde{\psi}\rangle \in \mathrm{SR}_k \end{aligned}$$

Same problem in disguise!

Application to matrix product states (MPS):

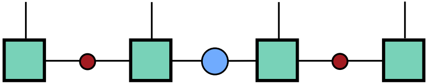


What is the best low-Schmidt-rank approximation to an entangled state, $|\psi\rangle$?

$$\begin{aligned} \text{maximize} \quad & F(|\psi\rangle, |\tilde{\psi}\rangle) \\ \text{subject to} \quad & |\tilde{\psi}\rangle \in \mathrm{SR}_k \end{aligned}$$

Same problem in disguise!

Application to matrix product states (MPS):

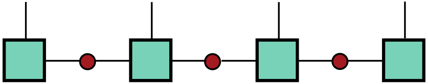


What is the best low-Schmidt-rank approximation to an entangled state, $|\psi\rangle$?

$$\begin{aligned} \text{maximize} \quad & F(|\psi\rangle, |\tilde{\psi}\rangle) \\ \text{subject to} \quad & |\tilde{\psi}\rangle \in \mathrm{SR}_k \end{aligned}$$

Same problem in disguise!

Application to matrix product states (MPS):



What is the best low-Schmidt-rank approximation to an entangled state, $|\psi\rangle$?

$$
\begin{aligned}
\text{maximize} \quad & F(|\psi\rangle, |\tilde{\psi}\rangle) \\
\text{subject to} \quad & |\tilde{\psi}\rangle \in \mathrm{SR}_k
\end{aligned}
$$

Same problem in disguise!

Application to matrix product states (MPS):

VOLUME 93, NUMBER 4 PHYSICAL REVIEW LETTERS week ending 23 JULY 2004

Efficient Simulation of One-Dimensional Quantum Many-Body Systems

Guifré Vidal

Institute for Quantum Information, California Institute of Technology, Pasadena, California 91125, USA

(Received 10 February 2004; published 19 July 2004)

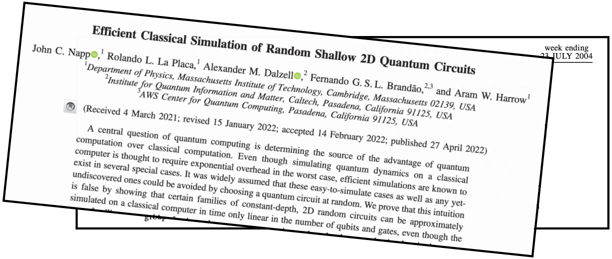
We present a numerical method to simulate the time evolution, according to a generic Hamiltonian made of local interactions, of quantum spin chains and systems alike. The efficiency of the scheme depends on the amount of entanglement involved in the simulated evolution. Numerical analysis indicates that this method can be used, for instance, to efficiently compute time-dependent properties of low-energy dynamics in sufficiently regular but otherwise arbitrary one-dimensional quantum many-body systems. As by-products, we describe two alternatives to the density matrix renormalization group method.

What is the best *low-Schmidt-rank* *approximation to an* *entangled* *state,* $|\psi\rangle$*?*

$$\begin{aligned} \text{maximize} \quad & F(|\psi\rangle, |\tilde{\psi}\rangle) \\ \text{subject to} \quad & |\tilde{\psi}\rangle \in \mathrm{SR}_k \end{aligned}$$

Same problem in disguise!

Application to matrix product states (MPS):



week ending
23 JULY 2004

Efficient Classical Simulation of Random Shallow 2D Quantum Circuits

John C. Napp,[1] Rolando L. La Placa,[1] Alexander M. Dalzell,[2] Fernando G. S. L. Brandão,[2,3] and Aram W. Harrow[1]

[1]*Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA*
[2]*Institute for Quantum Information and Matter, Caltech, Pasadena, California 91125, USA*
[3]*AWS Center for Quantum Computing, Pasadena, California 91125, USA*

(Received 4 March 2021; revised 15 January 2022; accepted 14 February 2022; published 27 April 2022)

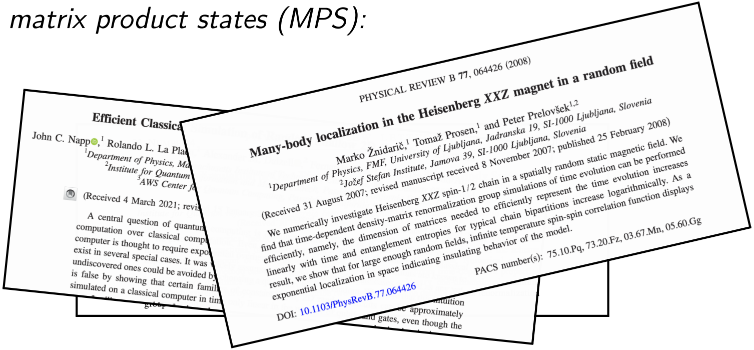
A central question of quantum computing is determining the source of the advantage of quantum computation over classical computation. Even though simulating quantum dynamics on a classical computer is thought to require exponential overhead in the worst case, efficient simulations are known to exist in several special cases. It was widely assumed that these easy-to-simulate cases as well as any yet-undiscovered ones could be avoided by choosing a quantum circuit at random. We prove that this intuition is false by showing that certain families of constant-depth, 2D random circuits can be approximately simulated on a classical computer in time only linear in the number of qubits and gates, even though the

What is the best low-Schmidt-rank approximation to an entangled state, $|\psi\rangle$?

$$\begin{aligned} \text{maximize} \quad & F(|\psi\rangle, |\tilde{\psi}\rangle) \\ \text{subject to} \quad & |\tilde{\psi}\rangle \in \mathrm{SR}_k \end{aligned}$$

Same problem in disguise!

Application to matrix product states (MPS):



What is the best low-Schmidt-rank approximation to an entangled state, $|\psi\rangle$?

$$\begin{aligned} \text{maximize} \quad & F(|\psi\rangle, |\tilde{\psi}\rangle) \\ \text{subject to} \quad & |\tilde{\psi}\rangle \in \mathrm{SR}_k \end{aligned}$$

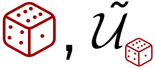
Same problem in disguise!

Application to matrix product states (MPS):

Classical randomness helps when approximating quantum processes

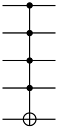
$$\mathcal{U} \longrightarrow \left(\text{🎲}, \tilde{\mathcal{U}}_{\text{🎲}} \right)$$

Classical randomness helps when approximating quantum processes

$\mathcal{U} \longrightarrow \big($  $, \tilde{\mathcal{U}}\big)$

Examples:

- *n*-qubit Toffoli [GKZ'25]

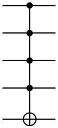


Classical randomness helps when approximating quantum processes

$$\mathcal{U} \longrightarrow \left(\quad , \tilde{\mathcal{U}} \right)$$

Examples:

- n-qubit Toffoli [GKZ'25]

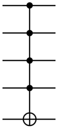


Classical randomness helps when approximating quantum processes

$\mathcal{U} \longrightarrow \Big($  $, \tilde{\mathcal{U}}$  $\Big)$

Examples:

- *n*-qubit Toffoli [GKZ'25]
- error mitigation [TBG'17]

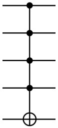


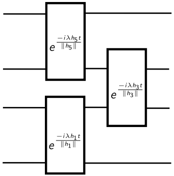
Classical randomness helps when approximating quantum processes

$$\mathcal{U} \longrightarrow \Big($$  $$, \tilde{\mathcal{U}}$$  $$\Big)$$

Examples:

- *n*-qubit Toffoli [GKZ'25]
- error mitigation [TBG'17]
- qDRIFT [Cam'18]



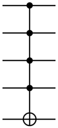


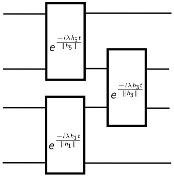
Classical randomness helps when approximating quantum processes

$$\mathcal{U} \longrightarrow \left(\text{🎲}, \tilde{\mathcal{U}}_{\text{🎲}}\right)$$

Examples:

- n-qubit Toffoli [GKZ'25] $\longrightarrow$ fewer T-gates
- error mitigation [TBG'17] $\longrightarrow$ ...noise-free ops.
- qDRIFT [Cam'18] $\longrightarrow$ ...Trotter steps



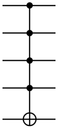


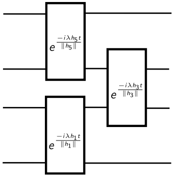
Classical randomness helps when approximating quantum processes

$$\mathcal{U} \longrightarrow \left(\quad , \tilde{\mathcal{U}} \right)$$

Examples:

- n-qubit Toffoli [GKZ'25] $\longrightarrow$ fewer T-gates
- error mitigation [TBG'17] $\longrightarrow$ ...noise-free ops.
- qDRIFT [Cam'18] $\longrightarrow$ ...Trotter steps

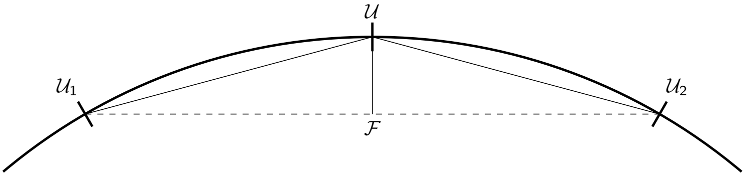




Why?

Classical randomness helps when approximating quantum processes

Reason 1: *Convex hull*



Classical randomness helps when approximating quantum processes

Reason 2: *Two-player game*

$$\|\mathcal{U} - \mathcal{F}\|_\diamond = \max_{\rho, M} \operatorname{tr}\left\{ M\left[(\mathcal{U} \otimes I)(\rho) - (\mathcal{F} \otimes I)(\rho) \right] \right\}$$

Classical randomness helps when approximating quantum processes

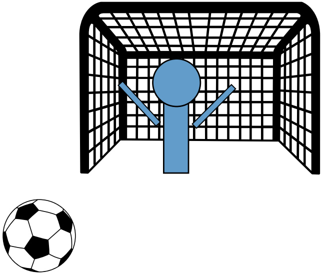
Reason 2: *Two-player game*

$$\min_{\mathcal{F}} \|\mathcal{U} - \mathcal{F}\|_\diamond = \min_{\mathcal{F}} \max_{\rho, M} \mathrm{tr}\Big\{ M \big[(\mathcal{U} \otimes I)(\rho) - (\mathcal{F} \otimes I)(\rho) \big] \Big\}$$

Classical randomness helps when approximating quantum processes

Reason 2: *Two-player game*

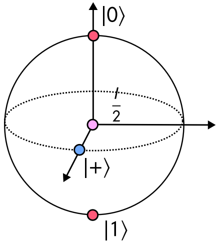
$$\min_{\mathcal{F}} \|\mathcal{U} - \mathcal{F}\|_{\diamond} = \min_{\mathcal{F}} \max_{\rho, M} \mathrm{tr}\Big\{ M \big[(\mathcal{U} \otimes I)(\rho) - (\mathcal{F} \otimes I)(\rho) \big] \Big\}$$



Q: Does randomness help for truncation?

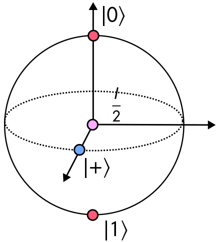
Q: Does randomness help for truncation?

- Truncate $|+\rangle\langle+|$ to a *1-sparse* state



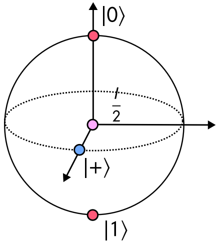
Q: Does randomness help for truncation?

- Truncate $|+\rangle\langle+|$ to a *1-sparse* state
- We get $T(|+\rangle, |0\rangle) = \frac{1}{\sqrt{2}}$, and similarly for $|1\rangle$



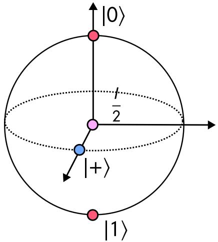
Q: Does randomness help for truncation?

- Truncate $|+\rangle\langle+|$ to a *1-sparse* state
- We get $T(|+\rangle, |0\rangle) = \frac{1}{\sqrt{2}}$, and similarly for $|1\rangle$
- But $T(|+\rangle, I/2) = \frac{1}{2}\| I/2 - |+\rangle\langle+| \|_1 = 1/2$



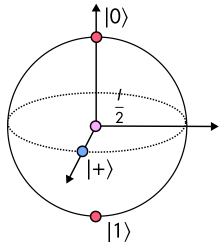
Q: Does randomness help for truncation?

- Truncate $|+\rangle\langle+|$ to a *1-sparse* state
- We get $T(|+\rangle, |0\rangle) = \frac{1}{\sqrt{2}}$, and similarly for $|1\rangle$
- But $T(|+\rangle, I/2) = \frac{1}{2}\|I/2 - |+\rangle\langle+|\|_1 = 1/2$
- So error $\sqrt{\varepsilon} \to \varepsilon$



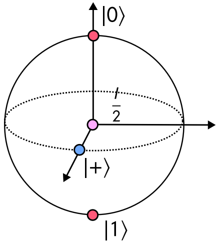
Q: Does randomness help for truncation?

- Truncate $|+\rangle\langle+|$ to a *1-sparse* state
- We get $T(|+\rangle, |0\rangle) = \frac{1}{\sqrt{2}}$, and similarly for $|1\rangle$
- But $T(|+\rangle, I/2) = \frac{1}{2}\||I/2 - |+\rangle\langle+|\|_1 = 1/2$
- So error $\sqrt{\varepsilon} \to \varepsilon$



Q: Does randomness help for truncation?

- Truncate $|+\rangle\langle+|$ to a *1-sparse* state
- We get $T(|+\rangle, |0\rangle) = \frac{1}{\sqrt{2}}$, and similarly for $|1\rangle$
- But $T(|+\rangle, I/2) = \frac{1}{2}\| I/2 - |+\rangle\langle+| \|_1 = 1/2$
- So error $\sqrt{\varepsilon} \to \varepsilon$



A: Yes.

(FvDG, pure) $\quad 1 - F^2 \leq T \leq \sqrt{1 - F^2}$

Quadratic Advantage

- If $\sqrt{\varepsilon} \equiv \sqrt{1 - F^2}$ is the best `dtrunc` TD, the best `rtrunc` TD is

$$\text{(FvDG, pure)} \qquad \varepsilon \leq T \leq \sqrt{\varepsilon}, \quad \varepsilon \equiv 1 - F^2$$

Quadratic Advantage

- If $\sqrt{\varepsilon} \equiv \sqrt{1-F^2}$ is the best `dtrunc` TD, the best `rtrunc` TD is

$$\text{(FvDG, pure)} \qquad \varepsilon \leq T \leq \sqrt{\varepsilon}, \quad \varepsilon \equiv 1 - F^2$$

- *When does quadratic advantage* $\sqrt{\varepsilon} \to \varepsilon$ *occur?*

Quadratic Advantage

- If $\sqrt{\varepsilon} \equiv \sqrt{1 - F^2}$ is the best `dtrunc` TD, the best `rtrunc` TD is

$$\text{(FvDG, pure)} \qquad \varepsilon \leq T \leq \sqrt{\varepsilon}, \quad \varepsilon \equiv 1 - F^2$$

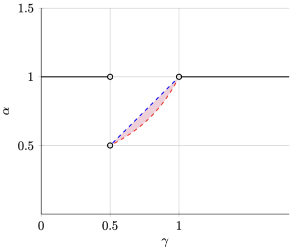
- *When does quadratic advantage* $\sqrt{\varepsilon} \to \varepsilon$ *occur?*
- Let $\psi_j \propto j^{-\gamma}$ for $\gamma \geq 0$. For large dimensions, $T \sim \varepsilon^{\alpha}$:

Quadratic Advantage

- If $\sqrt{\varepsilon} \equiv \sqrt{1-F^2}$ is the best `dtrunc` TD, the best `rtrunc` TD is

$$\text{(FvDG, pure)} \qquad \varepsilon \leq T \leq \sqrt{\varepsilon}, \quad \varepsilon \equiv 1 - F^2$$

- *When does quadratic advantage $\sqrt{\varepsilon} \to \varepsilon$ occur?*
- Let $\psi_j \propto j^{-\gamma}$ for $\gamma \geq 0$. For large dimensions, $T \sim \varepsilon^{\alpha}$:



So far: (i) *truncating quantum states is important*, & (ii) *randomization often helps quadratically*

Q: Can we solve for the optimal randomized truncation?

So far: (i) *truncating quantum states is important*, & (ii) *randomization often helps quadratically*

Q: Can we solve for the optimal randomized truncation?

I.e., we want

Algorithm 1.

Given: $|\psi\rangle$, k, $\|\cdot\|$

Output: $\{(p_x, |\varphi_x\rangle)\}_x$ such that

1. $|\varphi_x\rangle \in \mathrm{SR}_k\ \forall x$, and
2. $\rho = \sum_x p_x |\varphi_x\rangle\langle\varphi_x|$ minimizes distance to $|\psi\rangle$ in $\|\cdot\|$

So far: (i) *truncating quantum states is important*, & (ii) *randomization often helps quadratically*

Q: Can we solve for the optimal randomized truncation?

I.e., we want

Algorithm 1.

Given: $|\psi\rangle$, k, $\|\cdot\|$

Output: $\{(p_x, |\varphi_x\rangle)\}_x$ such that

1. $|\varphi_x\rangle \in \mathrm{SR}_k \; \forall x$, and
2. $\rho = \sum_x p_x |\varphi_x\rangle\langle\varphi_x|$ minimizes distance to $|\psi\rangle$ in $\|\cdot\|$

Let's focus on worst-case error, *trace distance*: $T(\rho, \sigma) := \frac{1}{2}\|\rho - \sigma\|_1$

Caution:

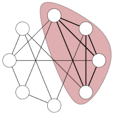
1. Deciding if a mixed state $\rho \in \mathrm{Sep}$ is NP-hard.
2. Let $\mathcal{I}_k := \mathrm{conv}\{|w\rangle\langle w| : \| |w\rangle \|_0 \leq k\}$. We prove the following theorem:

[1] Here, $\mathcal{S}_k := \mathrm{conv}\{|w\rangle\langle w| : |w\rangle \in \mathrm{SR}_k\}$.

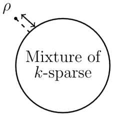
Caution:

1. Deciding if a mixed state $\rho \in \mathrm{Sep}$ is NP-hard.
2. Let $\mathcal{I}_k := \mathrm{conv}\{|w\rangle\langle w| : \||w\rangle\|_0 \leq k\}$. We prove the following theorem:

Theorem (this work, [JLRS'25]). *It is* NP*-hard to decide if* $\rho \in \mathcal{I}_k$.



$\leq$

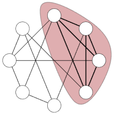


[1]Here, $\mathcal{S}_k := \mathrm{conv}\{|w\rangle\langle w| : |w\rangle \in \mathrm{SR}_k\}$.

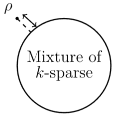
Caution:

1. Deciding if a mixed state $\rho \in \mathrm{Sep}$ is NP-hard.
2. Let $\mathcal{I}_k := \mathrm{conv}\{|w\rangle\langle w| : \||w\rangle\|_0 \leq k\}$. We prove the following theorem:

Theorem (this work, [JLRS'25]). *It is* NP*-hard to decide if* $\rho \in \mathcal{I}_k$.



$\leq$



... so don't expect a general algorithm that solves $\min_{\sigma \in \mathcal{I}_k} T(\rho, \sigma)$ or $\min_{\sigma \in \mathcal{S}_k} T(\rho, \sigma)$[1] for mixed states.

[1] Here, $\mathcal{S}_k := \mathrm{conv}\{|w\rangle\langle w| : |w\rangle \in \mathrm{SR}_k\}$.

Main Result

For a given *pure* state $|\psi\rangle \in \mathbb{C}^d$ we give *efficient* algorithms to *solve for* the optimal randomized truncation in trace distance (or robustness).

Main Result

For a given *pure* state $|\psi\rangle \in \mathbb{C}^d$ we give *efficient* algorithms to *solve for* the optimal randomized truncation in trace distance (or robustness).

- *solve for* means:
  1. compute $T^\star := \min_{\sigma \in \mathcal{I}_k} T(|\psi\rangle, \sigma)$
  2. compute an optimal $\sigma^\star \in \mathcal{I}_k$ in the standard basis
  3. sample $\{(p_x, |\varphi_x\rangle)\}$ such that $\sigma^\star = \sum_x p_x |\varphi_x\rangle\langle\varphi_x|$.

Main Result

For a given *pure* state $|\psi\rangle \in \mathbb{C}^d$ we give *efficient* algorithms to *solve for* the optimal randomized truncation in trace distance (or robustness).

- *solve for* means:
  1. compute $T^\star := \min_{\sigma \in \mathcal{I}_k} T(|\psi\rangle, \sigma)$
  2. compute an optimal $\sigma^\star \in \mathcal{I}_k$ in the standard basis
  3. sample $\{(p_x, |\varphi_x\rangle)\}$ such that $\sigma^\star = \sum_x p_x |\varphi_x\rangle\langle\varphi_x|$.
- *efficient* means: poly(d) time.

How to Randomly Truncate

Assume $|\psi\rangle \in \mathbb{R}^d$

How to Randomly Truncate

Assume $|\psi\rangle \in \mathbb{R}^d$

- k-`sparse dtrunc`:

$$|\tilde{\psi}\rangle \propto \psi_1 |1\rangle + \cdots + \psi_k |k\rangle$$

How to Randomly Truncate

Assume $|\psi\rangle \in \mathbb{R}^d$

- k-`sparse dtrunc`:

$$|\tilde{\psi}\rangle \propto \psi_1 |1\rangle + \cdots + \psi_k |k\rangle$$

- k-`sparse unbiased`:

$$|\tilde{\psi}\rangle \propto \psi_1 |1\rangle + \cdots + \psi_{k-r-1} |k-r-1\rangle + \theta\big(|j_1\rangle + \cdots + |j_{r+1}\rangle\big),$$

$$\text{where } S \equiv (j_1, \ldots, j_{r+1}) \subset \{k-r, \ldots, d\}, \ \Pr_{S\sim p}[j \in S] = \frac{\psi_j}{\theta}$$

  - so $\mathbb{E}_{S\sim p}|\tilde{\psi}\rangle = |\psi\rangle$
  - freedom to pick r

How to Randomly Truncate

Assume $|\psi\rangle \in \mathbb{R}^d$

- k-`sparse rtrunc`:

$$|\tilde{\psi}\rangle \propto \psi_1 |1\rangle + \cdots + \psi_{k-r-1} |k-r-1\rangle + \theta\big(|j_1\rangle + \cdots + |j_{r+1}\rangle\big),$$

$$\text{where } S \equiv (j_1, \ldots, j_{r+1}) \subset \{k-r, \ldots, \ell - 1\}, \;\; \Pr_{S\sim p}[j \in S] = \frac{\psi_j}{\theta} - \lambda$$

How to Randomly Truncate

Assume $|\psi\rangle \in \mathbb{R}^d$

- k-`sparse rtrunc`:

$$|\tilde{\psi}\rangle \propto \psi_1 |1\rangle + \cdots + \psi_{k-r-1} |k-r-1\rangle + \theta\big(|j_1\rangle + \cdots + |j_{r+1}\rangle\big),$$

$$\text{where } S \equiv (j_1, \ldots, j_{r+1}) \subset \{k-r, \ldots, \ell - 1\}, \ \Pr_{S\sim p}[j \in S] = \frac{\psi_j}{\theta} - \lambda$$

  - ▷ then $\sigma^\star = \mathbb{E}_{S\sim p}|\tilde{\psi}\rangle\langle\tilde{\psi}|$ satisfies $T^\star = T(|\psi\rangle, \sigma^\star)$

How to Randomly Truncate

Assume $|\psi\rangle \in \mathbb{R}^d$

- k-sparse `rtrunc`:

$$|\tilde{\psi}\rangle \propto \psi_1 |1\rangle + \cdots + \psi_{k-r-1} |k-r-1\rangle + \theta\big(|j_1\rangle + \cdots + |j_{r+1}\rangle\big),$$

$$\text{where } S \equiv (j_1, \ldots, j_{r+1}) \subset \{k-r, \ldots, \ell - 1\}, \ \Pr_{S\sim p}[j \in S] = \frac{\psi_j}{\theta} - \lambda$$

  - then $\sigma^\star = \mathbb{E}_{S\sim p} |\tilde{\psi}\rangle\langle\tilde{\psi}|$ satisfies $T^\star = T(|\psi\rangle, \sigma^\star)$
  - look for the correct $r, \ell \quad \longrightarrow \quad$ determines λ

How to Sample Indices

Represent using $x \in \{0, 1\}^n$ with $|x| = s$ for $1 \leq s \leq n - 1$.

How to Sample Indices

Represent using $x \in \{0,1\}^n$ with $|x| = s$ for $1 \leq s \leq n-1$.

Problem: distribution has support on $\binom{n}{s}$ strings...

How to Sample Indices

Represent using $x \in \{0,1\}^n$ with $|x| = s$ for $1 \leq s \leq n-1$.

Problem: distribution has support on $\binom{n}{s}$ strings...

Solution: use a Gibbs distribution

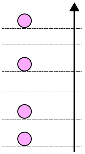
How to Sample Indices

Represent using $x \in \{0,1\}^n$ with $|x| = s$ for $1 \le s \le n-1$.

Problem: distribution has support on $\binom{n}{s}$ strings...

Solution: use a Gibbs distribution

$$p_\mu(x) = \frac{e^{-\langle x,\mu\rangle}}{Z_\mu}, \quad Z_\mu = \sum_x e^{-\langle x,\mu\rangle}$$



"s non-interacting fermions in n energy levels $\mu_1, \ldots, \mu_n$."

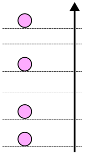
How to Sample Indices

Represent using $x \in \{0,1\}^n$ with $|x| = s$ for $1 \le s \le n-1$.

Problem: distribution has support on $\binom{n}{s}$ strings...

Solution: use a Gibbs distribution

$$p_\mu(x) = \frac{e^{-\langle x,\mu\rangle}}{Z_\mu}, \quad Z_\mu = \sum_x e^{-\langle x,\mu\rangle}$$



"s non-interacting fermions in n energy levels $\mu_1, \ldots, \mu_n$."

Known algorithms to:

- compute Z_μ, $q(\mu) := \Pr_{x\sim p_\mu}[x_j = 1]$, and two-element marginals [CDL'94, Til'06]
- sample from p_μ
  - we give an improved algorithm in this work, $O(ns)$ improved to $O(n \log n)$

we know the *right* marginals $q_j := \Pr_{x \sim p^\star}[x_j = 1]$ (recall $\frac{\psi_j}{\theta} - \lambda$)

Q: how to find the right $\mu \in \mathbb{R}^n$, given $q \in \mathbb{R}^n$?

A: solve for $\mu^\star$ minimizing $g_q(\mu)$.

$$g_q(\mu) := \langle q, \mu \rangle + \log(Z_\mu)$$

we know the *right* marginals $q_j := \Pr_{x \sim p^\star}[x_j = 1]$ (recall $\frac{\psi_j}{\theta} - \lambda$)

Q: how to find the right $\mu \in \mathbb{R}^n$, given $q \in \mathbb{R}^n$?

A: solve for $\mu^\star$ minimizing $g_q(\mu)$.

$\longrightarrow$ *Why do this?*

$$g_q(\mu) := \langle q, \mu \rangle + \log(Z_\mu)$$

we know the *right* marginals $q_j := \Pr_{x \sim p^\star}[x_j = 1]$ (recall $\frac{\psi_j}{\theta} - \lambda$)

Q: how to find the right $\mu \in \mathbb{R}^n$, given $q \in \mathbb{R}^n$?

A: solve for $\mu^\star$ minimizing $g_q(\mu)$.

$\longrightarrow$ *Why do this?*

$$g_q(\mu) := \langle q, \mu \rangle + \log(Z_\mu) = \mathrm{H}(p^\star) + \mathrm{D}(p^\star \| p_\mu)$$

we know the *right* marginals $q_j := \Pr_{x \sim p^\star}[x_j = 1]$ (recall $\frac{\psi_j}{\theta} - \lambda$)

Q: how to find the right $\mu \in \mathbb{R}^n$, given $q \in \mathbb{R}^n$?

A: solve for $\mu^\star$ minimizing $g_q(\mu)$.

$\longrightarrow$ *Why do this?* ✓

$$g_q(\mu) := \langle q, \mu \rangle + \log(Z_\mu) = \mathrm{H}(p^\star) + \mathrm{D}(p^\star \| p_\mu)$$

we know the *right* marginals $q_j := \Pr_{x \sim p^\star}[x_j = 1]$ (recall $\frac{\psi_j}{\theta} - \lambda$)

Q: how to find the right $\mu \in \mathbb{R}^n$, given $q \in \mathbb{R}^n$?

A: solve for $\mu^\star$ minimizing $g_q(\mu)$.

$\longrightarrow$ *Why do this?* ✓

$\longrightarrow$ *Can we do this?*

$$g_q(\mu) := \langle q, \mu \rangle + \log(Z_\mu) = \mathrm{H}(p^\star) + \mathrm{D}(p^\star \| p_\mu)$$

we know the *right* marginals $q_j := \Pr_{x \sim p^\star}[x_j = 1]$ (recall $\frac{\psi_j}{\theta} - \lambda$)

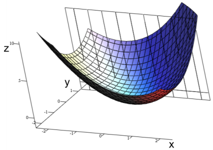
Q: how to find the right $\mu \in \mathbb{R}^n$, given $q \in \mathbb{R}^n$?

A: solve for $\mu^\star$ minimizing $g_q(\mu)$.

$\longrightarrow$ *Why do this?* ✓

$\longrightarrow$ *Can we do this?*

$$g_q(\mu) := \underset{\text{linear}}{\underset{\downarrow}{\langle q, \mu \rangle}} + \underset{\text{convex}}{\underset{\downarrow}{\log(Z_\mu)}} = \mathrm{H}(p^\star) + \mathrm{D}(p^\star \| p_\mu)$$



we know the *right* marginals $q_j := \Pr_{x \sim p^\star}[x_j = 1]$ (recall $\frac{\psi_j}{\theta} - \lambda$)

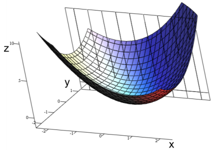
Q: how to find the right $\mu \in \mathbb{R}^n$, given $q \in \mathbb{R}^n$?

A: solve for $\mu^\star$ minimizing $g_q(\mu)$.

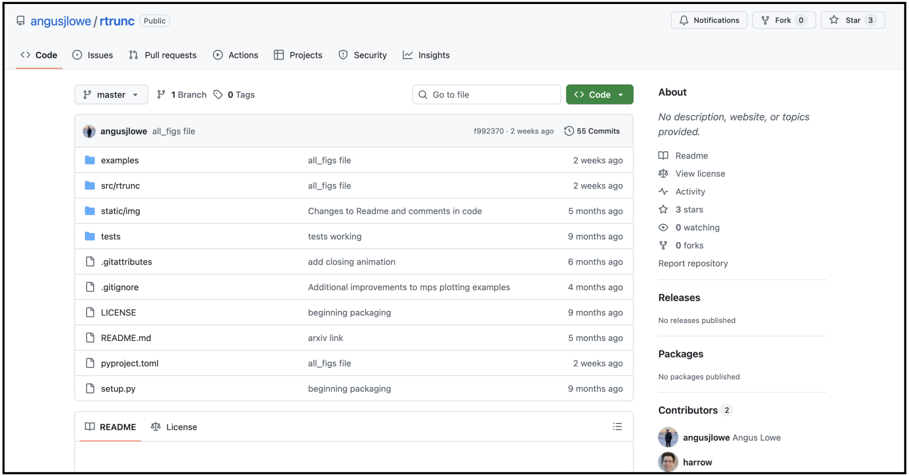
$\longrightarrow$ *Why do this?* ✓

$\longrightarrow$ *Can we do this?* ✓

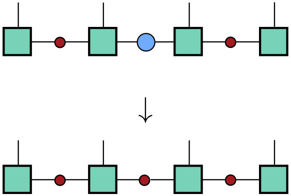
$$g_q(\mu) := \underbrace{\langle q, \mu \rangle}_{\text{linear}} + \underbrace{\log(Z_\mu)}_{\text{convex}} = \mathrm{H}(p^\star) + \mathrm{D}(p^\star \| p_\mu)$$

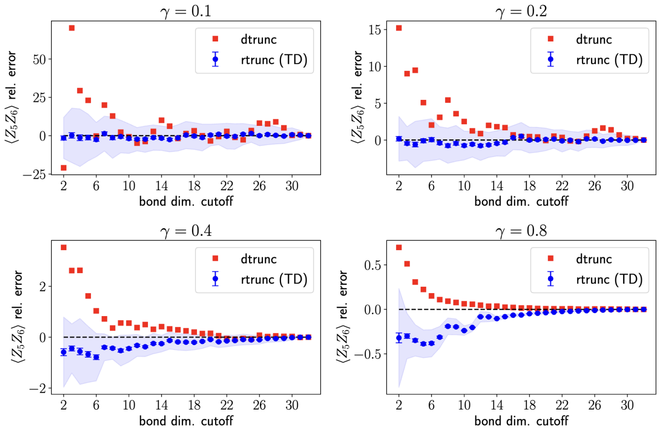


...and gradient is $\nabla_\mu g_q(\mu) = q - q(\mu)$


angusjlowe / rtrunc Public
Notifications
Fork 0
Star 3
Code
Issues
Pull requests
Actions
Projects
Security
Insights
master
1 Branch
0 Tags
Go to file
Code
angusjlowe all_figs file
f992370 · 2 weeks ago
55 Commits
examples all_figs file 2 weeks ago
src/rtrunc all_figs file 2 weeks ago
static/img Changes to Readme and comments in code 5 months ago
tests tests working 9 months ago
.gitattributes add closing animation 6 months ago
.gitignore Additional improvements to mps plotting examples 4 months ago
LICENSE beginning packaging 9 months ago
README.md arxiv link 5 months ago
pyproject.toml all_figs file 2 weeks ago
setup.py beginning packaging 9 months ago
README
License
About
No description, website, or topics provided.
Readme
View license
Activity
3 stars
0 watching
0 forks
Report repository
Releases
No releases published
Packages
No packages published
Contributors 2
angusjlowe Angus Lowe
harrow

Numerics





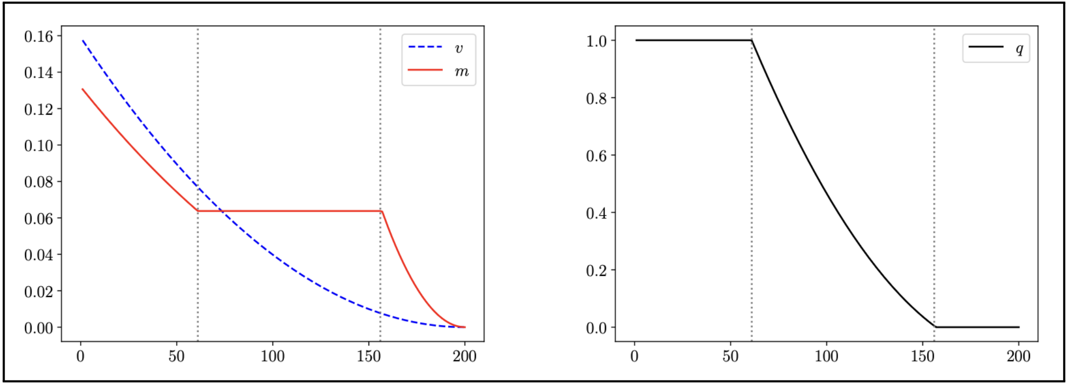
$\gamma = 0.1$
$\gamma = 0.2$
$\gamma = 0.4$
$\gamma = 0.8$
dtrunc
rtrunc (TD)
$\langle Z_5 Z_6 \rangle$ rel. error
bond dim. cutoff

Optimality Proof Sketch

$$T^\star = \min_{\sigma \in \mathcal{I}_k} \frac{1}{2} \| |\psi\rangle\langle\psi| - \sigma \|_1 = \min_{\sigma \in \mathcal{I}_k} \max_{0 \preceq M \preceq I} \operatorname{tr}\left(M(|\psi\rangle\langle\psi| - \sigma)\right).$$

Whiteboard...

Optimality Proof Sketch



Outlook

Main Result: we solve for the optimal randomized truncation to a quantum state.

Some future directions:

- More applications
  - Systematic numerics for MPS, time evolution of quantum many-body systems
- Improved runtime guarantees
  - Suspect $O(d^3)$ time suffices (comparable to SVD)
- Other metrics besides trace distance and robustness?
- *Minimax perspective to compute optimal randomized unitary synthesis*
- *Application beyond quantum states?*

The Approximation of One Matrix by Another of Lower Rank
C. Eckart and G. Young
Psychometrika, 1(3) (1936)

Multi-qubit Toffoli with exponentially fewer T gates
D. Gosset, R. Kothari, and C. Zhang
preprint (2025)

Error mitigation for short-depth quantum circuits
K. Temme, S. Bravyi, and J. M. Gambetta
PRL Vol. 119 Iss. 18 (2017)

Cat Icon
Shannon B.
Noun Project (CC BY 3.0)

Dice Icon
leo-graph.com
Noun Project (CC BY 3.0)

Soccer Net
Haven Kirby
Noun Project (CC BY 3.0)

Soccer Ball
sm kang
Noun Project (CC BY 3.0)